Public-key cryptography in Tor and pluggable transports

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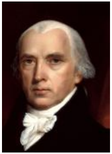
Tor

Attend Roger's talk on Friday.

Motivation

Sender
"Jefferson"

Network
"Eavesdropper"



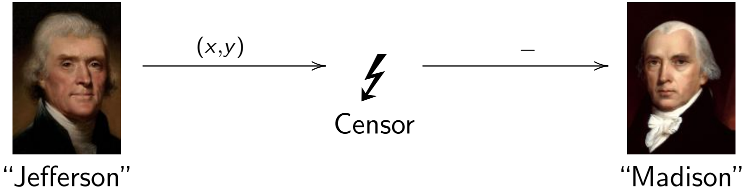
Receiver
"Madison"

Motivation #1 Channels are spying on our (meta-)data.

Motivation #2 Channels are modifying our (meta-)data.

Motivation #3 Channels interrupt and block suspicious communication.

DH key exchange



- Censor wants to block Tor (or whatever) traffic.
- Censor knows that Tor uses curve $E : y^2 = x^3 + ax + b$ over finite field $\mathbb{F}_p$.
- Jefferson sends (x, y) on E.
- Censor intercepts message, parses it as two field elements, checks whether (x, y) is a point on E. If so, break connection.
- Hasse's theorem says there are around p points on E over $\mathbb{F}_p$; that's very small compared to p^2 pairs. Random chance $1/p$.

DH key exchange

“Jefferson” $\xrightarrow{x_1,x_2,x_3\ldots}$ $\qquad$ $\xrightarrow{x_1,x_2,x_3}$ “Madison”

Censor

- Jefferson sends x, belonging to (x, y) on E.
- Each connection starts with a DH handshake, so there are several x_i.
- Censor intercepts message, parses it as one field element, checks whether x_i belongs to a point (x_i, y_i) on E.
  If so sufficiently often, break connection.
- Hasse's theorem says there are around p points on E over $\mathbb{F}_p$.
  Most come in pairs $(x, \pm y)$.
- About half of all values in $\mathbb{F}_p$ appear as x-coordinates.
- Random chance $1/2^n$ after n messages.
- This ignores p not being a power of 2, e.g. worse for $p = 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

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- But once traffic looks uniformly random (symmetric crypto has a much easier time on this) it can be steganographically layered on top of “accepted” communication.
- Needed for Telex (Wustrow, Wolchok, Goldberg, and Halderman; USENIX 2011) and StegoTorus (Weinberg, Wang, Yegneswaran, Briesemeister, Cheung, Wang, and Boneh; ACM CCS 2012).

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- Needed also for kleptography (exfiltrating keys to the adversary), e.g. Young and Yung SCN 2010.

How to use the idea

- Let $S \subseteq \{0,1\}^t$. Here: $S \subseteq \mathbb{F}_p$.
- Want map $\iota : S \to E(S)$ and inverse (limited to set $\iota(S)$).
- Want ι and ι^{-1} be efficiently computable and $\iota(S)$ be large in $E(\mathbb{F}_p)$, e.g. cover about half of all points.
- In DH, Jefferson picks j, computes jP. If $jP \notin \iota(S)$ he picks a new j. He sends $\iota^{-1}(jP)$. Same for Madison.
  On average 2 tries, only in local computation.
- In Schnorr signatures, signer Bob has public key $\tau_B = \iota^{-1}(bP)$ and private key b.
  To sign m, the sender picks random r until $rP \in \iota(S)$, computes $\tau = \iota^{-1}(rP), h = H(\tau||\tau_B||m)$, $s = r + hb \pmod{\ell}$.
  The signature is (τ, s).
- Signature verification:
  Compute $bP = \iota(\tau_B), rP = \iota(\tau), h = H(\tau||\tau_B||m)$.
  Compare $rP + h(bP)$ and sP.
  This works: $sP = (r + hb)P = rP + h(bP)$.

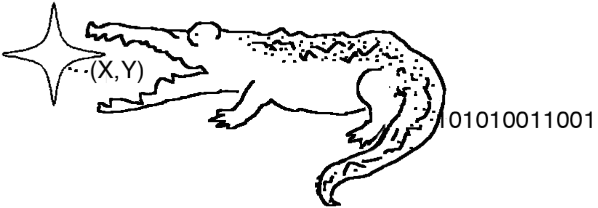
Two approaches ... and their shortcomings

Assume that p is close to power of 2.

- Hash strings to curve points; increment till valid x-coordinate is found.
  - Points can have multiple preimages.
  - Points can have no preimages.
  - Really hard to get uniform distribution (reject with probability proportional to the number of preimages? How many are there? How to get deterministic map?).
  - Finding all the preimages means point counting.
- Use curve E and its quadratic twist E'.
  - Each $x \in \mathbb{F}_p$ belongs to two points: $(x, \pm y)$ on E, $(x, \pm y)$ on E' or $(x, 0)$ on both curves.
  - Get uniformity by switching to right curve.
  - Requires two keys for everything (doubles key size).
  - Problems with parties choose non-matching curves in DH.

Elligator!

Joint work with Bernstein, Hamburg, and Krasnova (CCS 2013).



We use slightly different curve shape.

$$y^2 = x^3 + Ax^2 + Bx$$

with $AB(A^2 - 4B) \neq 0$ (usually $A = 0$ included but not here).

- This curve has a point $(0, 0)$ of order 2.
- For $B = 1$ called *Montgomery curve* (can have C in Cy^2).
- Tor uses Curve25519 in ntor for building circuits (see Friday?). Curve25519 is a Montgomery curve with $A = 486662$ and $p = 2^{255} - 19$.

Elligator

- Rewrite curve equation as $y^2 = x(x^2 + Ax + B)$.
- Find two values x_1, x_2 such that

$$x_1^2 + Ax_1 + B = x_2^2 + Ax_2 + B \text{ and } x_1/x_2 \neq \square.$$

- In finite fields we have $\not\square \cdot \not\square = \square$, so either x_1 or x_2 belongs to an (x, y) on the curve (except for $y = 0$),
- Transform equality into $x_1 + x_2 = -A$ (i.e. $x_1 = -A - x_2$).
- Let $x_1/x_2 = ur^2$, where u is a fixed non-square in $\mathbb{F}_p$.
- Combine to $(-A - x_2)/x_2 = ur^2$, i.e. $x_2 = -A/(1 + ur^2)$ and $x_1 = -Aur^2/(1 + ur^2)$.
- This defines map $\iota(r) = (x_1, \sqrt{x_1(x_1^2 + Ax_1 + B)})$ or $\iota(r) = (x_2, -\sqrt{x_2(x_1^2 + Ax_1 + B)})$ (pick the one defined).

Inverse map

- $\iota(S)$ is the set of $(x, y) \in E(\mathbb{F}_p)$ with
  - $x \neq -A$,
  - if $y = 0$ then $x = 0$, and
  - $-ux(x + A) = \square$.
- If $(x, y) \in \iota(S)$ then $\bar{r} \in S$ is defined and $\iota(\bar{r}) = (x, y)$:

$$\bar{r} = \begin{cases} \sqrt{-x/((x+A)u)} & \text{if } y \in \sqrt{\mathbb{F}_p^2}; \\ \sqrt{-(x+A)/(ux)} & \text{if } y \notin \sqrt{\mathbb{F}_p^2}. \end{cases}$$

Application to Curve25519

Here $q \equiv 1 \pmod 4$ and $u = 2$ is a non-square.
Need to specify a square-root function for $\mathbb{F}_p$.

- Given a square $a \in \mathbb{F}_p$, compute $b = a^{(q+3)/8}$.
  (Note that $q \equiv 5 \pmod 8$, so $(q+3)/8$ is an integer.)
  Then $b^4 = a^2$, i.e., $b^2 \in \{a, -a\}$.
- Define $\sqrt{a}$ as $|b|$ if $b^2 = a$ and as $\left|b\sqrt{-1}\right|$ otherwise.
- Here $|b|$ means b if $b \in \{0, 1, \ldots, (q-1)/2\}$, otherwise $-b$.

Cost of computing ι:

- 1 square-root computation,
- 1 inversion,
- 1 computation of square-root selection
- a few multiplications.

Note that the inversion and the square-root computation can be combined into one exponentiation,

More motivation

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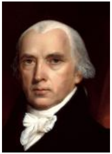

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Motivation #4 Network nodes want to know how many of them exist.

Hidden services/onion services

- For better protection against eavesdropping, users can reach facebook at `https://facebookcorewwwi.onion`.
- This means their traffic never leaves the Tor network.
- Facebook advertises their .onion page, so their existence is public.
- Other public .onion pages are xmpp servers for chat.
- Reasons for private .onion sites
  - Use Tor to deal with stupid network configuration (e.g. at TU/e).
  - Local chat services using Ricochet.
  - Collaborative servers (small group, not public).
  - File sharing, online shops, ...
  - Secure drop sites.
- General idea is that nobody knows all the existing sites.
- See Roger's talk for more details.

Related keys

- Alice has secret key a and public key $A = aP$ on elliptic curve.
- These are known to people she wants to connect with.
- Alice's server changes location every day and there are Directory Services (DS) providing locations based on keys.

Related keys

- Alice has secret key a and public key $A = aP$ on elliptic curve.
- These are known to people she wants to connect with.
- Alice's server changes location every day and there are Directory Services (DS) providing locations based on keys.
- DSs are used randomly, but all servers will likely come by in a month, so for fixed keys the directory knows all servers.
- Alice goes to a conference and doesn't want to bring a, but throw-away keys A' for each day, but
  - She doesn't want to get a new certificate for A'.
  - She doesn't want to distribute new public keys.
  - She wants to be able to decrypt after the trip, but not keep old a'.
- Idea (Zooko Wilcox-O'Hearn; Gregory Maxwell; Robert Ransom; Christian Grothoff):
  If $d = H(date)$ is public, anybody can compute $A + dP$ or dA which are public keys for $a + d$ or ad.
- Put $d = H(date, A)$, for d secret from those not knowing A.
- Also used in Bitcoin (BIP 32), Tahoe-LAFS, and GNUNet.

How to use this idea?

- Make .onion addresses harder to harvest by directory servers (Tor track # 8106).
- DSs store information on location of A under the key A, along with a signature under A.
- Alice can produce signatures under A' from having da.
- There is no authority limiting the number of keys and servers. Of course anybody can submit a fake entry B with a signature for its alleged location under B.
- But: nobody other than Alice can produce signature under A'.
- Recall Schnorr signatures: Signature on m is (R, s) with $R = rP, h = H(R||A||m)$, $s = r + ha \pmod{\ell}$.
  Verification:
  Compute $h = H(R||A||m)$ and compare $R + h(A)$ and sP.

How to use this idea?

- Toss in some more: make $d = H(date||P||A)$.
- DS receives location date for server A' with signature under A' using $a' = da$. Checks signature and stores information.
- Authorized client computes A' from A and date; asks DS for information on A'.
- Client verifies signature on information obtained from DS, using A'.
- Verification can use precomputed A' or include extra d in equations.
- A bit more tricky in practice to deal with Ed25519, which has nontrivial cofactors.
- This involves lots of non-standard crypto assumptions and modeling.