Code-based cryptography for secure communication

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  but in practice we do not use public-key crypto to encrypt a message.
- We encrypt a short symmetric key and use that to encrypt the actual message.

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- A key-encapsulation mechanism requires 3 algorithms:
  1. Key generation, generating a public-key private-key pair.
  2. Encapsulation, taking a public key, producing key k and ciphertext. k is then used in symmetric crypto.
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- Can think of DH as a KEM:

$$\mathrm{KEM} - \mathrm{Enc}(g^a) = (g^{ra}, g^r) = (k, c)$$

- Anna-Lena Horlemann explained Niederreiter for encryption.
- Niederreiter as KEM takes public key, picks random vector of length n, weight t.

How does TLS (`https`) work?

Client | **Server**

$(\mathsf{sk}_C, \mathsf{pk}_C) \leftarrow\$ \mathsf{KGen}$

$$\xrightarrow{\mathsf{pk}_C}$$

$(\mathsf{sk}_S, \mathsf{pk}_S) \leftarrow\$ \mathsf{KGen}$

$k \leftarrow \mathsf{DH}(\mathsf{sk}_S, \mathsf{pk}_C)$

$$\xleftarrow{\mathsf{pk}_S}$$

$k \leftarrow \mathsf{DH}(\mathsf{sk}_C, \mathsf{pk}_S)$

$$\xrightarrow[\text{proves } C \text{ knows } k]{\text{stuff encrypted using } k}$$

$\Sigma \leftarrow \mathsf{Sig}(\text{everything sent so far})$

this uses a long-term signing key

$$\xleftarrow[\text{stuff encrypted using } k]{\Sigma}$$

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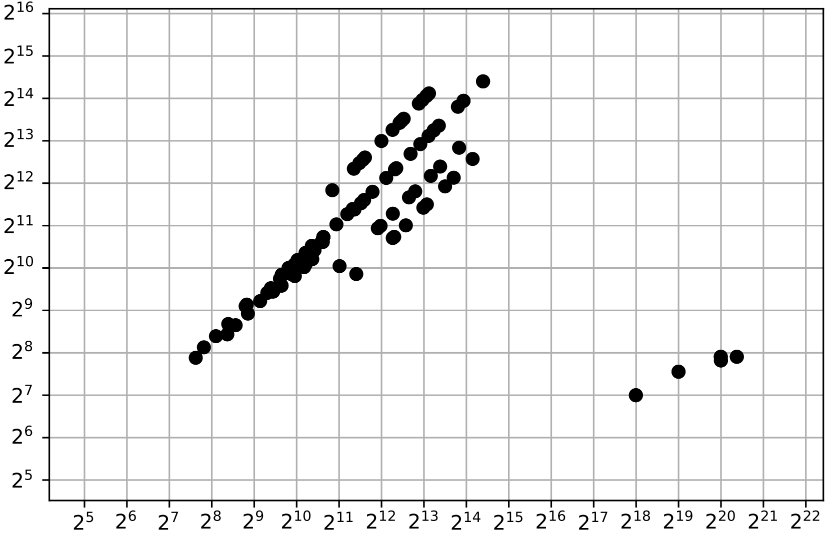
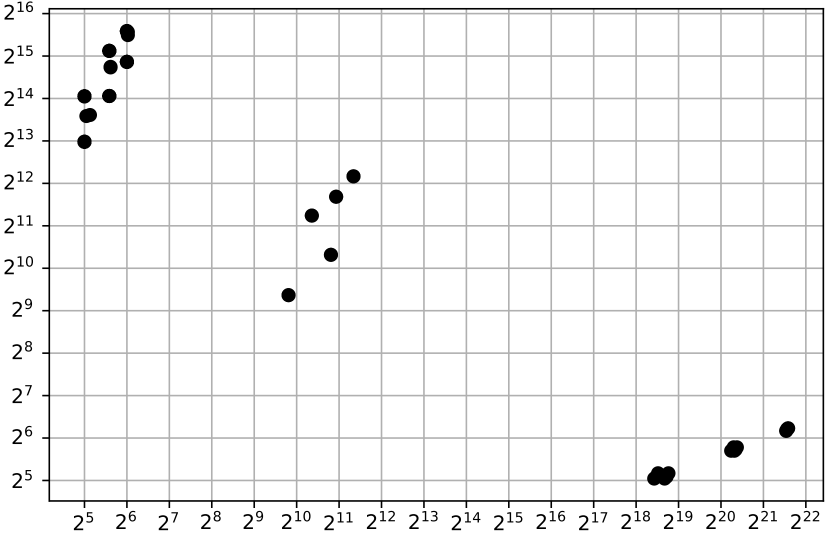
  ensure that as strong as strongest not as weak as weakest.
- New security assumptions, new proofs, lots of new code.

Encryption (KEM): ciphertext size (vertical) vs. public-key size (horizontal)



Signatures: signature size (vertical) vs. public-key size (horizontal)



Deployment issues & solutions

- Different recommendations for rollout in different risk scenarios:
  - Use most efficient systems with ECC or RSA, to ease usage and gain familiarity.
  - Use most conservative systems (possibly with ECC), to ensure that data really remains secure.
- Protocol integration and implementation problems:
  - Key sizes or message sizes are larger for post-quantum systems, but IPv6 guarantees only delivery of $\leq$ 1280-byte packets, TLS software has length limits, etc.
  - Google experimented with larger keys and noticed delays and dropped connections.
  - Long-term keys require extra care (reaction attacks).
- Some libraries exist, quality is getting better.
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NIST PQC submission Classic McEliece

No patents.
Shortest ciphertexts.
Fast open-source constant-time software implementations.
Very conservative system, expected to last; has strongest security track record.

Sizes with similar post-quantum security to AES-128, AES-192, AES-256:

Metric	**mceliece348864**	**mceliece460896**	**mceliece6960119**
Public-key size	261120 bytes	524160 bytes	1047319 bytes
Secret-key size	6452 bytes	13568 bytes	13908 bytes
Ciphertext size	128 bytes	188 bytes	226 bytes
Key-generation time	52415436 cycles	181063400 cycles	417271280 cycles
Encapsulation time	43648 cycles	77380 cycles	143908 cycles
Decapsulation time	130944 cycles	267828 cycles	295628 cycles

See `https://classic.mceliece.org` for authors, details & parameters.

Key issues for McEliece

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BIG PUBLIC KEYS.

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(DoS = Denial of Service)

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Our goal: Eliminate these attacks by eliminating all per-client storage on server.

Goodness, what big keys you have!

Public keys look like this:

$$K = \begin{pmatrix} 1 & 0 & \dots & 0 & 1 & \dots & 1 & 0 & 1 \\ 0 & 1 & \dots & 0 & 0 & \dots & 0 & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & 1 & \dots & 1 & 1 & 0 \\ 0 & 0 & \dots & 1 & 0 & \dots & 1 & 1 & 1 \end{pmatrix}$$

Left part is $(n-k) \times (n-k)$ identity matrix (no need to send).
Right part is random-looking $(n-k) \times k$ matrix.
E.g. $n = 6960$, $k = 5413$, so $n - k = 1547$.

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Encryption xors secretly selected columns, e.g.

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Can servers avoid storing big keys?

$$K = \begin{pmatrix} 1 & 0 & \dots & 0 & 1 & \dots & 1 & 0 & 1 \\ 0 & 1 & \dots & 0 & 0 & \dots & 0 & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & 1 & \dots & 1 & 1 & 0 \\ 0 & 0 & \dots & 1 & 0 & \dots & 1 & 1 & 1 \end{pmatrix} = (I_{n-k}|K')$$

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Receive columns of K' one at a time, store and update partial sum.

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On the real Internet, without per-client state:
Don't reveal intermediate results!
Which columns are picked is the secret message!
Intermediate results show whether a column was used or not.

McTiny

Partition key

$$K' = \begin{pmatrix} K_{1,1} & K_{1,2} & K_{1,3} & \dots & K_{1,\ell} \\ K_{2,1} & K_{2,2} & K_{2,3} & \dots & K_{2,\ell} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ K_{r,1} & K_{r,2} & K_{r,3} & \dots & K_{r,\ell} \end{pmatrix}$$

- Each submatrix $K_{i,j}$ small enough to fit (including header) into network packet.
- Client feeds the $K_{i,j}$ to server & handles storage for the server.
- Server computes $K_{i,j}e_j$, puts result into cookie.
- Cookies are encrypted by server to itself using some temporary symmetric key (same key for all server connections).
  No per-client memory allocation.
- Cookies also encrypted & authenticated to client.
- Client sends several $K_{i,j}e_j$ cookies, receives their combination.
- More stuff to avoid replay & similar attacks.

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- Cookies also encrypted & authenticated to client.
- Client sends several $K_{i,j}e_j$ cookies, receives their combination.
- More stuff to avoid replay & similar attacks.
- Several round trips, but no per-client state on the server.

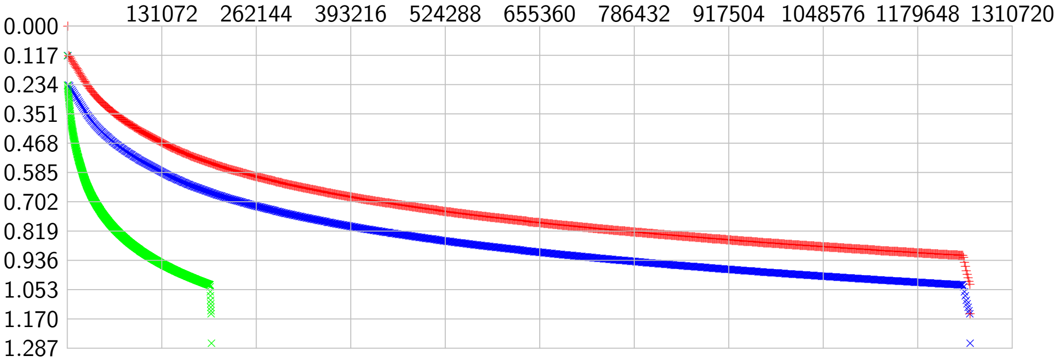
Packet sizes in each phase of mceliece6960119

phase		bytes/packet	packets	bytes
0	query	810	1	810
	reply	121	1	121
1	query	1226	952	1 167 152
	reply	140	952	133 280
2	query	1185	17	20 145
	reply	133	17	2 261
3	query	315	1	315
	reply	315	1	315
	queries		971	1 188 422
	replies		971	135 977

Entries count only application-layer data and not counting UDP/IP/Ethernet overhead.

A public key is 1 047 319 bytes.

Measurements of our software (`https://mctiny.org`)



Client time vs. bytes sent, bytes acknowledged, bytes in acknowledgments.
Curve shows packet pacing from our new user-level congestion-control library.

WireGuard

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- In WireGuard the server is known by a long-term DH key.
- This public key is exchanged out of band.

WireGuard

Client		**Server**
knows LTpk_S		has $\mathrm{LTsk}_S, \mathrm{LTpk}_S$

.... Actual start ....

$(\mathrm{sk}_C, \mathrm{pk}_C) \leftarrow_{\$} \mathrm{KGen}$

$k_1 \leftarrow \mathrm{DH}(\mathrm{sk}_C, \mathrm{LTpk}_S)$ $\quad \xrightarrow[\text{something with } k_1]{\mathrm{pk}_C} \quad$ $k_1 \leftarrow \mathrm{DH}(\mathrm{LTsk}_S, \mathrm{pk}_C), \text{check } k_1$

$(\mathrm{sk}_S, \mathrm{pk}_S) \leftarrow_{\$} \mathrm{KGen}$

$k_2 \leftarrow \mathrm{H}(k_1, \mathrm{DH}(\mathrm{sk}_S, \mathrm{pk}_C))$

$\xleftarrow[\text{something with } k_2]{\mathrm{pk}_S}$

$k_2 \leftarrow \mathrm{H}(k_1, \mathrm{DH}(\mathrm{sk}_C, \mathrm{pk}_S))$

$\xrightarrow[\text{or keys derived from } k_2]{\text{content encrypted with } k_2}$

'WireGuard' with KEMs

Client		**Server**
knows KEM LTpk_S		has KEM $\mathrm{LTsk}_S, \mathrm{LTpk}_S$

. . . . Actual start . . . .

$(\mathrm{sk}_C, \mathrm{pk}_C) \xleftarrow{\$} \mathrm{KGen}$		
$(k_1, c_1) \leftarrow \mathrm{KEM{-}Enc}(\mathrm{LTpk}_S)$	$\xrightarrow[\mathrm{Enc}(k_1, \mathrm{pk}_C)]{c_1}$	$k_1 \leftarrow \mathrm{KEM{-}Dec}(\mathrm{LTsk}_S, c_1)$
		$\mathrm{pk}_C \leftarrow \mathrm{Dec}(k_1, \mathrm{Enc}(k_1, \mathrm{pk}_C))$
		$(k_2', c_2) \leftarrow \mathrm{KEM{-}Enc}(\mathrm{pk}_C)$
	$\xleftarrow[\text{stuff to verify}]{c_2}$	$k_2 \leftarrow \mathrm{H}(k_1, k_2')$
$k_2' \leftarrow \mathrm{KEM{-}Dec}(\mathrm{sk}_C, c_2)$		
$k_2 \leftarrow \mathrm{H}(k_1, k_2')$	$\xrightarrow[\text{or keys derived from } k_2]{\text{content encrypted with } k_2}$	

Post-quantum WireGuard https://eprint.iacr.org/2020/379

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- Short-term KEM public key pk_C is sent and should be small.
- Post-quantum WireGuard uses Classic McEliece for the long-term KEM and lattice-based Saber for the short-term KEM.
- This showcases the small ciphertexts of Classic McEliece and does not notice the public-key size.

Different deployment strategy

PQConnect: An Automated Boring Protocol for Quantum-Secure Tunnels

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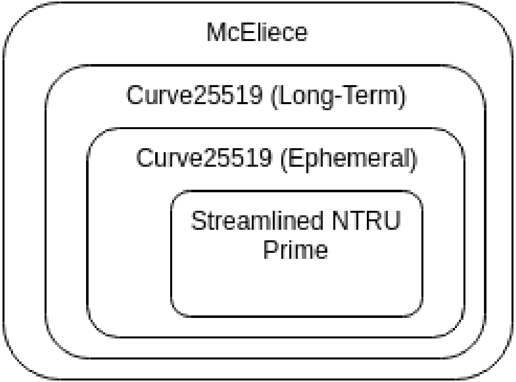
Different deployment strategy

PQConnect: An Automated Boring Protocol for Quantum-Secure Tunnels

- Do not patch PQC onto existing network protocols, but add a new layer with superior security.
- Can be gradually deployed.
- Add support for VPN-like tunnels to clients and servers but do this to the endpoints, not some intermediate VPN server.
- PQConnect is designed for security, handshake and ratcheting proven using Tamarin prover (formal verification tool).
- Use Curve25519 (pre-quantum) and Classic McEliece (conservative PQC) for long-term identity keys.
- Use Curve25519 (pre-quantum) and lattice-based Streamlined NTRU Prime (PQC) for ephemeral keys.

PQConnect handshake: Nesting schemes

Most conservative system on the outside.



Attacker can see long-term Curve25519 identity key,
can break it with a quantum computer,
but cannot obtain DH value as client's share is wrapped.

PQConnect handshake: Handling McElice keys

- McEliece is used for the long-term key, i.e., this key does not change.
- Store key for frequently visited sites (Google, Gmail, Facebook, Twitter,...)
- Link key download to obtaining IP address via DNS lookup.
  This is how the client know where to connect to. PQConnect piggy-backs on this with a hash of the key and info on where to download the key.

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  This is how the client know where to connect to. PQConnect piggy-backs on this with a hash of the key and info on where to download the key.
- Split key as in McTiny, download in small chunks and verify with hash;
  PQConnect also includes the Curve25519 key (256 bits, just a small corner).
- PQConnect benefits from small McEliece ciphertexts.
- Combine with lattice-based crypto for balance in ciphertext and public key size; security concerns alleviated by nesting.
- More information on protocol:
  `https://research.tue.nl/en/studentTheses/pqconnect`
  Paper and software still forthcoming.

Key ratchet advances by message and time

Complete protocol follows picture on previous slide.
All systems linked together to generate initial key c_0.
Keys are updated (ratcheted) to protect against later decryption by theft of computer equipment.
Immediately advance ratchet in 3 ways:

- New epoch master key: c_1.
- New branch keys: $c_{0,1}, c_{0,2}$.
- New message key: $c'_{0,1}$.

Delete key as soon as no longer needed.
Message keys can deal with delayed transmissions.

$$
\begin{tikzcd}
c_0 \arrow[dd] \arrow[dr] & & & & \\
& c_{0,1} \arrow[d] \arrow[dr] & & & \\
c_1 \arrow[dd] \arrow[dr] & c'_{0,1} & c_{0,2} \arrow[d] \arrow[dr] & & \\
& c_{1,1} \arrow[d] \arrow[dr] & c'_{0,2} & c_{0,3} \arrow[d] \arrow[dr] & \\
c_2 \arrow[dd] \arrow[dr] & c'_{1,1} & c_{1,2} \arrow[d] \arrow[dr] & c'_{0,3} & \ddots \\
& c_{2,1} \arrow[d] \arrow[dr] & c'_{1,2} & c_{1,3} \arrow[d] \arrow[dr] & \\
\vdots & c'_{2,1} & c_{2,2} \arrow[d] \arrow[dr] & c'_{1,3} & \ddots \\
& & c'_{2,2} & c_{2,3} \arrow[d] \arrow[dr] & \\
& & & c'_{2,3} & \ddots
\end{tikzcd}
$$

Further information

- https://pqcrypto.org our overview page.
- PQCrypto 2016, PQCrypto 2017, PQCrypto 2018, PQCrypto 2019, PQCrypto 2020, PQCrypto 2021 with many slides and videos online.
- https://pqcrypto.eu.org: PQCRYPTO EU Project.
  - PQCRYPTO recommendations.
  - Free software libraries (libpqcrypto, pqm4, pqhw).
  - Many reports, scientific articles, (overview) talks.
- YouTube channel Tanja Lange: Post-quantum cryptography.
- https://2017.pqcrypto.org/school: PQCRYPTO summer school with 21 lectures on video, slides, and exercises.
- https://2017.pqcrypto.org/exec and https://pqschool.org/index.html: Executive school (less math, more perspective).
- Quantum Threat Timeline from Global Risk Institute, 2019; 2021 update.
- Status of quantum computer development (by German BSI).
- NIST PQC competition.