Progress in Post-Quantum Cryptography

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History of post-quantum cryptography

- 2003 Daniel J. Bernstein introduces term Post-quantum cryptography.
- PQCrypto 2006: International Workshop on Post-Quantum Cryptography.
- ▶ PQCrypto 2008, PQCrypto 2010, PQCrypto 2011, PQCrypto 2013.
- ▶ 2014 EU publishes H2020 call including post-quantum crypto as topic. PQCRYPTO is funded.
- ▶ PQCrypto 2014.
- ▶ September 2015: Initial recommendations by PQCRYPTO.
- PQCrypto 2016.
- ▶ 2016: NIST announces competition for post-quantum systems.
- November 2017: Submissions for NIST competition due.
 PQCRYPTO submits 22 designs (out of a total of 69).
- ▶ April 2018: First NIST PQC proposer conference.



Algorithms for Quantum Computation: Discrete Logarithms and Factoring

Peter W. Shor AT&T Bell Labs Room 2D-149 600 Mountain Ave. Murray Hill, NJ 07974, USA

Abstract

A computer is generally considered to be a universal computational device; i.e., it is believed able to simulate any physical computational device with a cost in computation time of at most a polynomial factor. It is not clear whether this is still true when quantum mechanics is taken into consideration. Several researchers, starting with David Deutsch, have developed models for quantum mechanical computers and have investigated their computational properties. This paper gives Las Vegas algorithms for finding discrete logarithms and factoring integers on a quantum computer that take a number of steps which is polynomial in the input size, e.g., the number of digits of the integer to be factored. These two problems are generally considered hard on a classical computer and have been used as the basis of several proposed cryptosystems. (We thus give the first examples of quantum cryptanalysis.)

[1, 2]. Although he did not ask whether quantum mechanics conferred extra power to computation, he did show that a Turing machine could be simulated by the reversible unitary evolution of a quantum process, which is a necessary prerequisite for quantum computation. Deutsch [9, 10] was the first to give an explicit model of quantum computation. He defined both quantum Turing machines and quantum circuits and investigated some of their properties.

The next part of this paper discusses how quantum computation relates to classical complexity classes. We will thus first give a brief intuitive discussion of complexity classes for those readers who do not have this background. There are generally two resources which limit the ability of computers to solve large problems: time and space (i.e., memory). The field of analysis of algorithms considers the asymptotic demands that algorithms make for these resources as a function of the problem size. Theoretical computer scientists generally classify algorithms grows as a polynomial in the size of the input. The class of prob-



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- ▶ Also, Grover's algorithm speeds up brute-force searches.
- Example: Only 2^{64} quantum operations to break AES-128; 2^{128} quantum operations to break AES-256.



Even higher urgency for long-term confidentiality

- Attacker can break currently used encryption (ECC, RSA) with a quantum computer.
- ► Even worse, today's encrypted communication is being stored by attackers and will be decrypted years later with quantum computers. All data can be recovered in clear from recording traffic and breaking the public key scheme.
- How many years are you required to keep your data secret? From whom?





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- PQCRYPTO ICT-645622

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 but there will not be a public announcement ... and an important function of signatures is to protect operating system upgrades.
- ▶ Protect your upgrades *now* with post-quantum signatures.



Initial recommendations of long-term secure post-quantum systems

Daniel Augot, Lejla Batina, Daniel J. Bernstein, Joppe Bos, Johannes Buchmann, Wouter Castryck, Orr Dunkelman, Tim Güneysu, Shay Gueron, Andreas Hülsing, Tanja Lange, Mohamed Saied Emam Mohamed, Christian Rechberger, Peter Schwabe, Nicolas Sendrier, Frederik Vercauteren, Bo-Yin Yang



Initial recommendations

- ▶ **Symmetric encryption** Thoroughly analyzed, 256-bit keys:
 - ► AES-256
 - ► Salsa20 with a 256-bit key

Evaluating: Serpent-256, ...

- ▶ **Symmetric authentication** Information-theoretic MACs:
 - ▶ GCM using a 96-bit nonce and a 128-bit authenticator
 - ► Poly1305
- ▶ Public-key encryption McEliece with binary Goppa codes:
 - ▶ length n = 6960, dimension k = 5413, t = 119 errors

Evaluating: QC-MDPC, Stehlé-Steinfeld NTRU, ...

- ▶ Public-key signatures Hash-based (minimal assumptions):
 - XMSS with any of the parameters specified in CFRG draft
 - ► SPHINCS-256

Evaluating: HFEv-, ...



NIST Post-Quantum "Competition"

December 2016, after public feedback: NIST calls for submissions of post-quantum cryptosystems to standardize.

30 November 2017: NIST receives 82 submissions.

	Signatures	KEM/Encryption	Overall
Lattice-based	4	24	28
Code-based	5	19	24
Multi-variate	7	6	13
Hash-based	4		4
Other	3	10	13
Total	23	59	82



"Complete and proper" submissions

21 December 2017: NIST posts 69 submissions from 260 people.

BIG QUAKE. BIKE. CFPKM. Classic McEliece. Compact LWE. CRYSTALS-DILITHIUM. CRYSTALS-KYBER. DAGS. Ding Key Exchange. DME. DRS. DualModeMS. Edon-K. EMBLEM and R.EMBLEM. FALCON. FrodoKEM. GeMSS. Giophantus. Gravity-SPHINCS. Guess Again. Gui. HILA5. HiMQ-3. HK17. HQC. KINDI. LAC. LAKE. LEDAkem. LEDApkc. Lepton. LIMA. Lizard, LOCKER, LOTUS, LUOV, McNie, Mersenne-756839. MQDSS. NewHope. NTRUEncrypt. NTRU-HRSS-KEM. NTRU Prime. NTS-KEM. Odd Manhattan. OKCN/AKCN/CNKE. Ouroboros-R. Picnic. pqNTRUSign. pqRSA encryption. pqRSA signature. pgsigRM. QC-MDPC KEM. gTESLA. RaCoSS. Rainbow. Ramstake. RankSign. RLCE-KEM. Round2. RQC. RVB. SABER, SIKE, SPHINCS+, SRTPI, Three Bears, Titanium. Walnut DSA



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Some attack scripts already posted causing **total break** or **serious tweaks**. Many more receiving detailed analysis.



Classic McFliece conservative code-based cryptography

Daniel J. Bernstein, Tung Chou, Tanja Lange, Ingo von Maurich, Rafael Misoczki, Ruben Niederhagen, Edoardo Persichetti, Christiane Peters, Peter Schwabe, Nicolas Sendrier, Jakub Szefer, Wen Wang



Key sizes and key-generation speed

mceliece6960119 parameter set: 1047319 bytes for public key. 13908 bytes for secret key.

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Very fast in hardware (PQCrypto 2018; CHES 2017): a few million cycles at 231MHz using 129059 modules, 1126 RAM blocks on Altera Stratix V FPGA.



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Can tweak parameters for even smaller ciphertexts, not much penalty in key size.



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The McEliece system (with later key-size optimizations) uses $(c_0 + o(1))\lambda^2(\lg \lambda)^2$ -bit keys as $\lambda \to \infty$ to achieve 2^{λ} security against Prange's attack. Here $c_0 \approx 0.7418860694$.



40 years and more than 30 analysis papers later

1962 Prange; 1981 Clark-Cain, crediting Omura; 1988 Lee-Brickell; 1988 Leon; 1989 Krouk; 1989 Stern; 1989 Dumer; 1990 Coffey-Goodman; 1990 van

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https://pqcrypto.eu.org

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Replacing λ with 2λ stops all known *quantum* attacks.



Classic McEliece

McEliece's system prompted huge amount of followup work.

Some work improves efficiency while clearly preserving security:

- Niederreiter's dual PKE (use parity check matrix instead of generator matrix);
- many decoding speedups; . . .

Classic McEliece uses all this, with constant-time implementations.

- ▶ Write $H = (I_{n-k}|T)$, public key is $(n-k) \times k$ matrix T, $n-k = w \log_2 q$. H constructed from binary Goppa code.
- ightharpoonup Encapsulate using e of weight w.

mceliece6960119 parameter set (2008 Bernstein-Lange-Peters): $q=8192,\ n=6960,\ w=119.$

mceliece8192128 parameter set:

q = 8192, n = 8192, w = 128.



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- 6. There are no inversion failures for legitimate ciphertexts.



Classic McEliece highlights

- Security asymptotics unchanged by 40 years of cryptanalysis.
- Short ciphertexts.
- Efficient and straightforward conversion of OW-CPA PKE into IND-CCA2 KEM.
- ► Constant-time software implementations.
- FPGA implementation of full cryptosystem.
- ▶ Open-source (public domain) implementations.
- No patents.



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https://pqcrypto.eu.org

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12 fully broken (efficient script posted) systems fall into

- ► Codes: Edon-K, pqsigRM, RaCoSS, RankSign.
- ► Lattices: Compact LWE.
- Multivariate: CFPKM, DME.
- Other: Guess Again, HK17, RVB, SRTP, Walnut DSA.



Constructive progress – SPHINCS+

- Same as SPHINCS in terms of high level scheme design, but better few-time signatures.
- New protection against multi-target attacks.
- ▶ New few-time signature scheme FORS instead of HORST (different way of combining Merkle trees).
- ► Smaller signatures 30kB instead of 41kB or more signatures.
- Smaller public keys.
- Three versions (different hash functions)
 - SPHINCS+-SHA3 (using SHAKE256),
 - ► SPHINCS+-SHA2 (using SHA-256),
 - ► SPHINCS+-Haraka (using the Haraka short-input hash function).

https://pqcrypto.eu.org

See https://sphincs.org/ for more details.



Constructive progress – NTRUPrime

- Lattice-based encryption smaller public keys.
- Less structure for the attacker to use:
 - Computation is done modulo prime instead of modulo power of 2.
 - ▶ Rings change from using polynomial $x^n 1$ or $x^n + 1$ to $x^p x 1$, p prime.
 - ► No (nontrivial) subrings or fields.
- ▶ No decryption failures.

Metric	Streamlined	NTRU
	NTRU Prime 4591^{761}	LPRime 4591 ⁷⁶¹
Public-key size	1218 bytes	1047 bytes
Ciphertext size	1047 bytes	1175 bytes
Encapsulation time	59456 cycles	94508 cycles
Decapsulation time	97684 cycles	128316 cycles
Pre-quantum security	248 bits	225 bits

See https://ntruprime.cr.yp.to/ for more details.



Further resources

- ▶ https://2017.pqcrypto.org/school: PQCRYPTO summer school with 21 lectures on video + slides + exercises.
- https://2017.pqcrypto.org/exec: Executive school (12) lectures), less math, more overview. So far slides, soon videos.
- https://pqcrypto.org: Our survey site.
 - Many pointers: e.g., to PQCrypto conferences;
 - Bibliography for 4 major PQC systems.
- https://pgcrypto.eu.org: PQCRYPTO EU project.
 - Expert recommendations.
 - Free software libraries.
 - More video presentations, slides, papers.
- ▶ https://twitter.com/pqc_eu: PQCRYPTO Twitter feed.
- https://twitter.com/PQCryptoConf: PQCrypto conference Twitter feed.
- https://csrc.nist.gov/projects/ post-quantum-cryptography/round-1-submissions NIST PQC competition.

