Some Elliptic CUrve REsults

Elligator

by Daniel J. Bernstein, Mike Hamburg, Anna Krasnova, and Tanja Lange

MiniaLT

by W. Michael Petullo, Xu Zhang, Jon A. Solworth, Daniel J. Bernstein, and Tanja Lange

http://safecurves.cr.yp.to/

by Daniel J. Bernstein and Tanja Lange

December 3, 2013

Elligator

Your protocol sends elliptic curve points (public keys) but they need to look like random strings because

- you're fighting censorship; connection will be cut if your protocol is detected.

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http://elligator.cr.yp.to

Daniel J. Bernstein, Mike Hamburg, Anna Krasnova, and Tanja Lange

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You have been using an elliptic curve together with its twist for whatever reason.

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You might be interested in Elligator!

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Elligator

Choose prime $p = 2^n - c$ with small c, then integers $\leq (p-1)/2$ cover essentially all strings of $n-1$ bits.

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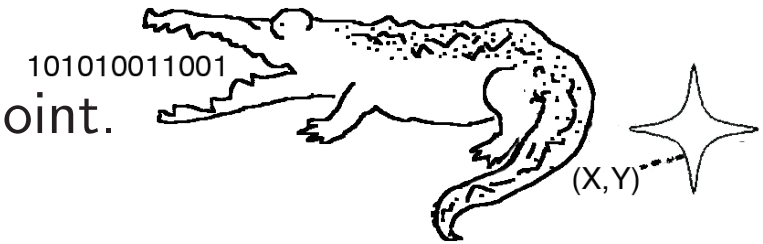
Choose prime $p = 2^n - c$ with small c, then integers $\leq (p-1)/2$ cover essentially all strings of $n-1$ bits.

Elligator map takes any integer $\leq (p-1)/2$ and produces curve point.



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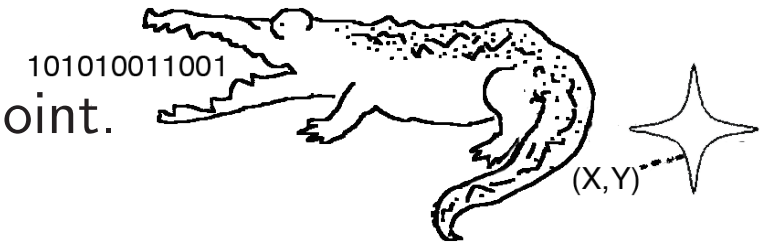
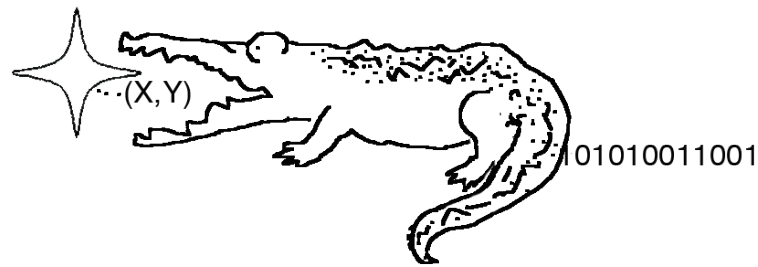


Inverse elligator map defined on $(p-1)/2$ points, produces string of $n-1$ bits.



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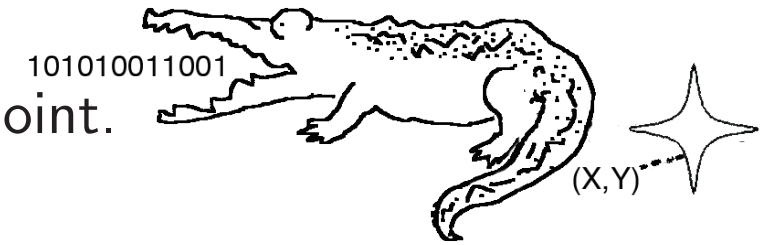

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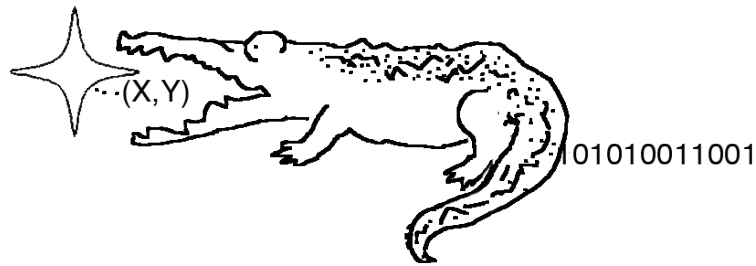

Maps are inverse of one another and efficiently computable.
Very fast test whether inverse map is defined on point
$\Rightarrow$ fast rejection of points, tweak protocol to try again.

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MinimaLT – Minimal Latency Tunneling

- MinimaLT protocol makes secure connections
- Replaces TLS and TCP; built on top of UDP
- Advantages:
  - Stronger than TLS

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http://www.ethos-os.org/

W. Michael Petullo, Xu Zhang, Jon A. Solworth, Daniel J. Bernstein, & me

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- Provides IP-address mobility: unlinkable, no handshake
- Overlaps ephemeral-key retrieval with DNS lookup
- Erases keys within minutes (TLS often takes months)
- Has strong protections against denial of service
- Fast crypto provided by NaCl.

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Extra advertisement

small (human-auditable) codebase: TweetNaCl

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SafeCurves:
choosing safe curves for elliptic-curve cryptography

All known security criteria for elliptic curves,
machine verified.

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All known security criteria for elliptic curves,
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Also: can the curve be backdoored?

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SafeCurves:
choosing safe curves for elliptic-curve cryptography

		Parameters:			ECDLP security:				ECC security:			
Curve	Safe?	field	equation	base	rho	transfer	disc	rigid	ladder	twist	complete	ind
Anomalous	False	True ✔	True ✔	True ✔	True ✔	False	False	True ✔	False	False	False	False
M-221	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔
E-222	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔
NIST P-224	False	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	False	False	False	False	False
Curve1174	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔
Curve25519	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔
BN(2,254)	False	True ✔	True ✔	True ✔	True ✔	False	False	True ✔	False	False	False	False
brainpoolP256t1	False	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	False	False	False	False
ANSSI FRP256v1	False	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	False	False	False	False	False
NIST P-256	False	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	False	False	True ✔	False	False
secp256k1	False	True ✔	True ✔	True ✔	True ✔	True ✔	False	True ✔	False	True ✔	False	False
M-383	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔
Curve383187	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔
brainpoolP384t1	False	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	False	True ✔	False	False
NIST P-384	False	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	False	False	True ✔	False	False
Curve3617	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔	True ✔

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