Quantum computing for cryptographers II

Gates and basic circuits

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idea and design by Daniel J. Bernstein

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SAC – Post-quantum cryptography

NOT gates

NOT_0 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(1, 3, 1, 4, 9, 5, 6, 2)$.

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NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
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$(5, 9, 2, 6, 3, 1, 4, 1)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state: NOT_0, swapping pairs.
Operation after measurement: flipping bit 0 of result.
Flip: output is not input.

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$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state: NOT_0, swapping pairs.
Operation after measurement: flipping bit 0 of result.
Flip: output is not input.

This slide shows the effect of NOT on our representation. This way we can simulate quantum computers to see whether algorithms work.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

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Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

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$(q_2,q_1,q_0) \mapsto (q_2,q_1,q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

Operation after measurement:
flipping bit 0 *if* bit 2 is set; i.e.,
$(q_2,q_1,q_0) \mapsto (q_2,q_1,q_0 \oplus q_2)$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
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e.g. C_2NOT_0:
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$(3, 1, 4, 1, 9, 5, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 2 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_2)$.

e.g. C_0NOT_2:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Operation after measurement:
flipping bit 2 *if* bit 0 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_0 \oplus q_2, q_1, q_0)$.

CNOT is its own inverse, thus it is reversible.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$: $(3, 1, 4, 1, 5, 9, 2, 6) \mapsto (3, 1, 4, 1, 5, 9, 6, 2)$.

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e.g. $C_2C_1NOT_0$: $(3,1,4,1,5,9,2,6) \mapsto (3,1,4,1,5,9,6,2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

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e.g. $C_2C_1NOT_0$: $(3, 1, 4, 1, 5, 9, 2, 6) \mapsto (3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $C_0C_1NOT_2$: $(3, 1, 4, 1, 5, 9, 2, 6) \mapsto (3, 1, 4, 6, 5, 9, 2, 1)$.

Toffoli gates

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e.g. $C_2C_1NOT_0$: $(3,1,4,1,5,9,2,6) \mapsto (3,1,4,1,5,9,6,2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $C_0C_1NOT_2$: $(3,1,4,1,5,9,2,6) \mapsto (3,1,4,6,5,9,2,1)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2 \oplus q_0 q_1, q_1, q_0)$.

Toffoli is its own inverse, thus it is reversible.

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

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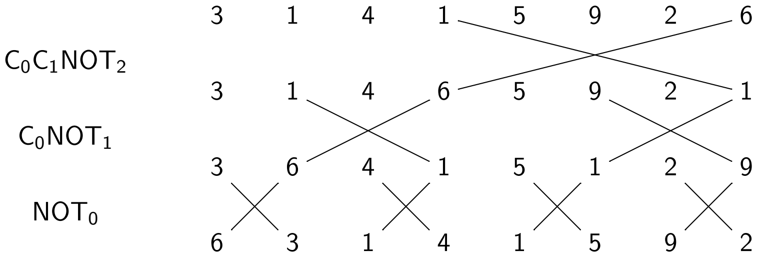
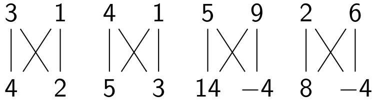
e.g. series of gates to rotate 8 positions by distance 1:



Hadamard gates

Hadamard_0:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard gates

Hadamard$_0$:

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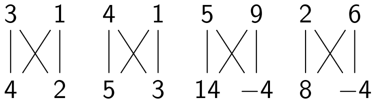
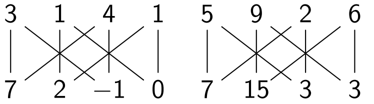


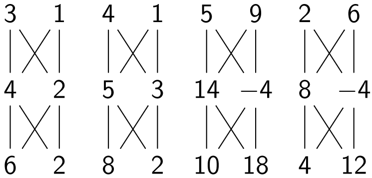
Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Some uses of Hadamard gates

Hadamard_0, Hadamard_0:



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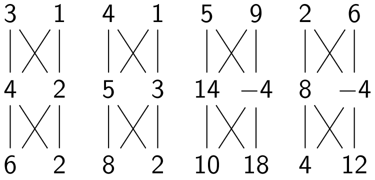
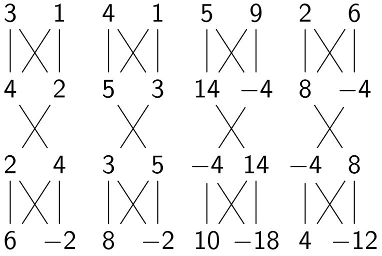


"Multiply each amplitude by 2."

This is not physically observable.
Disappears normally in scaling.
Hadamard is self inverse.

Hadamard$_0$, NOT$_0$, Hadamard$_0$:



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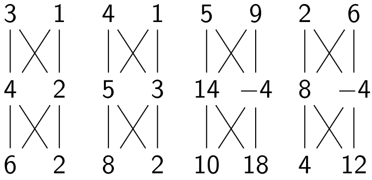

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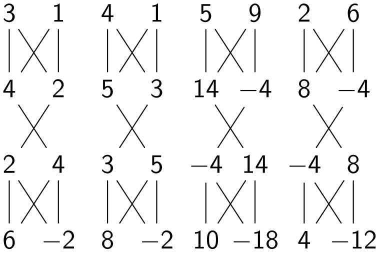
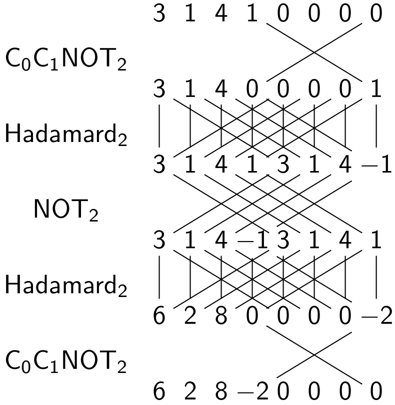


"Multiply each amplitude by 2,
Negate amplitude if q_0 is set."

No effect on measuring *now*.

Getting towards affecting measurements



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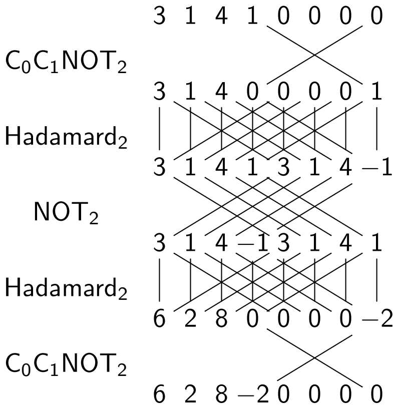

"Negate amplitude if q_0q_1 is set."
Assumes $q_2 = 0$: "ancilla" qubit.
Returns $q_2 = 0$ "clean".

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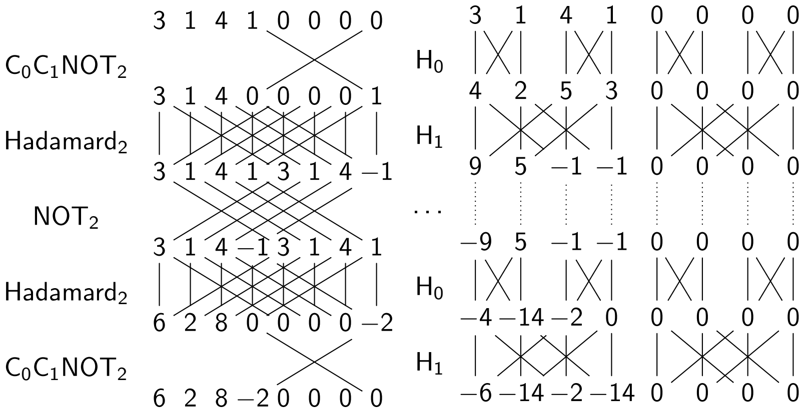

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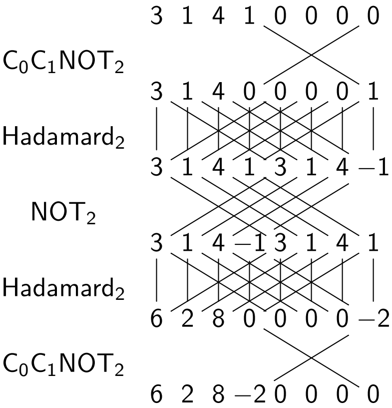

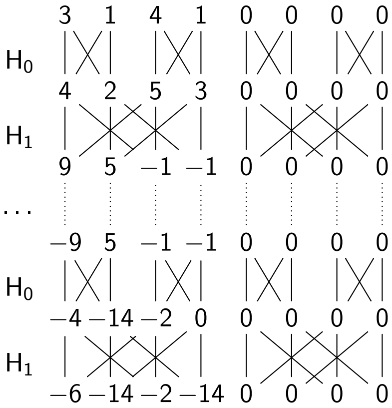

"Negate amplitude if $q_0 q_1$ is set."
Assumes $q_2 = 0$: "ancilla" qubit.
Returns $q_2 = 0$ "clean".

"Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.
This affects measurements.