# TECHNISCHE UNIVERSITEIT EINDHOVEN
## Faculty of Mathematics and Computer Science
## Exam Cryptography 1, Tuesday 27 January 2015

Name :

TU/e student number :

| Exercise | 1 | 2 | 3 | 4 | 5 | total |
|---|---|---|---|---|---|---|
| points | | | | | | |

**Notes:** Please hand in this sheet at the end of the exam. You may keep the sheet with the exercises.
This exam consists of 5 exercises. You have from 13:30 – 16:30 to solve them. You can reach 100 points.
Make sure to justify your answers in detail and to give clear arguments. Document all steps, in particular of algorithms; it is not sufficient to state the correct result without the explanation. If the problem requires usage of a particular algorithm other solutions will not be accepted even if they give the correct result.
All answers must be submitted on TU/e letterhead; should you require more sheets ask the proctor. State your name on every sheet.
Do not write in red or with a pencil.
You are allowed to use any books and notes, e.g. your homework. You are not allowed to use the textbooks of your colleagues.
You are allowed to use a calculator without networking abilities. Usage of laptops and cell phones is forbidden.

1. This problem is about ElGamal encryption in the group $\mathbb{F}_{2003}^*$ with generator $g = 5$.

   (a) Alice's public key is $h = 877$. Encrypt the message $m = 1002$ to Alice using ElGamal encryption with random value $k = 2^{10} + 1$. $\boxed{\text{4 points}}$

   (b) Charlie has private key $c = 123$. He receives ciphertext $(c_1, c_2) = (1410, 1815)$. Decrypt the message. $\boxed{\text{4 points}}$

2. This exercise is about computing discrete logarithms in some groups.

   (a) Alice and Bob use the **additive group** modulo $p = 1003$ with generator $g = 5$ for their Diffie-Hellman system. You observe the DH shares $a' = a \cdot g = 123$ and $b' = b \cdot g = 456$. Compute their shared secret. $\boxed{\text{4 points}}$

   (b) Use the baby-step-giant-step algorithm to determine Alice's secret key $a$ for the parameters in exercise 1, i.e. $\mathbb{F}_{2003}^*$, $g = 5$, and $h = 877$.
   Make sure to document all intermediate steps.
   $\boxed{\text{20 points}}$

3. This exercise is about factoring $n = 2015$. Obviously, 5 is a factor, so the rest of the exercise is about factoring the remaining factor $m = 2015/5 = 403$.

   (a) Use Pollard's rho method of factorization to find a factor of 403. Use starting point $x_0 = 2$, iteration function $x_{i+1} = x_i^2 + 1$ and Floyd's cycle finding method, i.e. compute $\gcd(x_{2i} - x_i, 403)$ until a non-trivial gcd is found. Make sure to document the intermediate steps.
   $\boxed{\text{8 points}}$

   (b) Perform one round of the Fermat test with base $a = 2$ to test whether 31 is prime.
   What is the answer of the Fermat test? $\boxed{\text{2 points}}$

   (c) Perform one round of the Miller-Rabin test with base $a = 2$ to test whether 31 is prime.
   What is the answer of the Miller-Rabin test? $\boxed{\text{4 points}}$

   (d) Use Dixon's factorization method to factor the number $n = 403$ using $a_1 = 22$. $\boxed{\text{6 points}}$

4. (a) Find all affine points on the Edwards curve $x^2 + y^2 = 1 + 2x^2y^2$ over $\mathbb{F}_{11}$. | 8 points |

   (b) Verify that $P = (3, 4)$ is on the curve. Compute the order of $P$. | 8 points |

   (c) Translate the curve **and** $P$ to Montgomery form

$$Bv^2 = u^3 + Au^2 + u.$$

| 4 points |

5. This exercise introduces the Paillier cryptosystem. Key generation works similar to that in RSA: Let $p$ and $q$ be large primes, put $n = pq, g = n+1$, and compute $\varphi(n) = (p-1)(q-1)$ and $\mu \equiv \varphi(n)^{-1} \bmod n$. The public key is $(n, g)$, the private key is $(\varphi(n), \mu)$.

   To encrypt message $m \in \mathbb{Z}/n$ pick a random $1 \le r < n$ with $\gcd(r, n) = 1$ and compute the ciphertext $c \equiv g^m \cdot r^n \bmod n^2$. Note the computation is done modulo $n^2$, not modulo $n$.

   To decrypt $c \in \mathbb{Z}/n^2$ compute $d \equiv c^{\varphi(n)} \bmod n^2$. Consider $d$ as an integer and observe that $d - 1$ is a multiple of $n$ (see below). Compute $e = (d - 1)/n$ and obtain the message as $m \equiv e\mu \bmod n$.

   (a) Encrypt the message 123 to a user with public key $(n, g) = (4087, 4088)$ using $r = 11$. | 2 points |

   (b) Your public key is $(n, g) = (3127, 3128)$ and your secret key is $(\varphi(n), \mu) = (3016, 2141)$. Decrypt the ciphertext $c = 8053838$. | 4 points |

   (c) Compute symbolically (no particular value of $n$ or $r$) $\varphi(n^2)$ and $r^{n\varphi(n)} \bmod n^2$, using $n = pq$. | 4 points |

   (d) Compute symbolically (no particular value of $n$ or $m$) $g^{m\varphi(n)} \bmod n^2$. | 4 points |

   (e) Explain why $d - 1$ is a multiple of $n$ and why decryption recovers $m$.
   **Hint: use the previous two parts.** | 4 points |

   (f) Let $c_1$ be the encryption of $m_1$ using some $r_1$ and let $c_2$ be the encryption of $m_2$ using some $r_2$, both for the same public key $(n, g)$. Show that $c \equiv c_1c_2 \bmod n^2$ decrypts to $m_1 + m_2$.
   Make sure to justify your answer. | 10 points |