Cryptographic hash functions II

Pollard rho for collision search

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2MMC10 – Cryptology

Generic hardness

The birthday paradox implies that if one draws elements at random from a set of m elements, then with 50% probability one has picked one element twice after about $\sqrt{\pi m/2}$ picks.
Hence it takes $O(2^{n/2})$ calls to H to find a collision.

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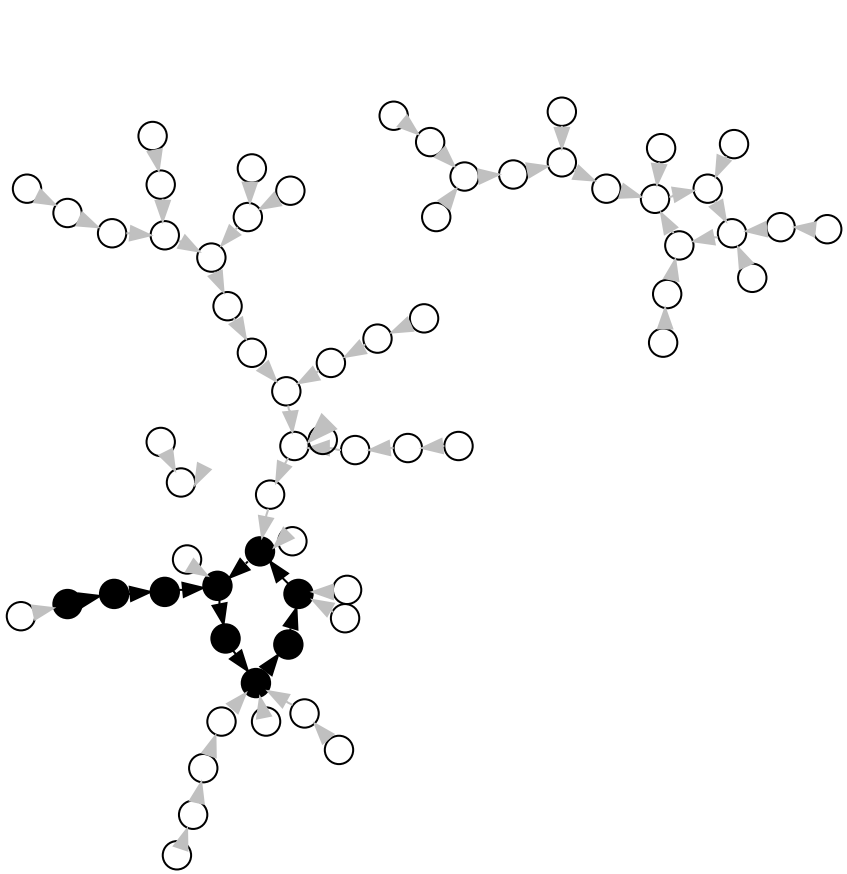
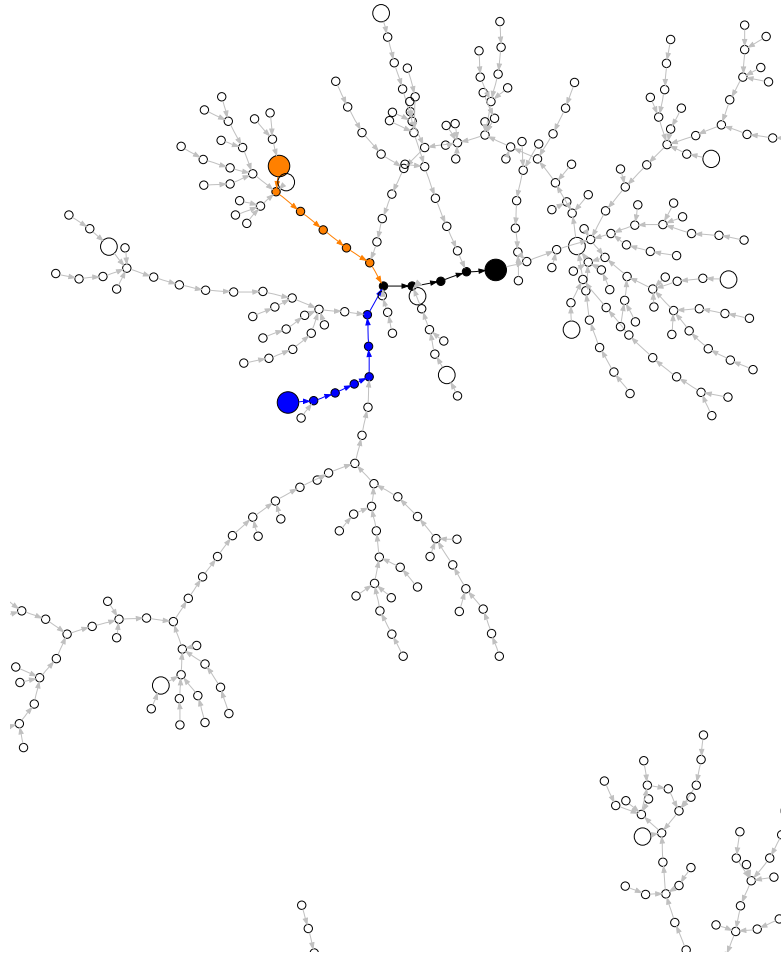
If message space includes output space, e.g., $H : \{0,1\}^* \to \{0,1\}^n$, use random starting point W_0 and iterate $W_{i+1} = H(W_i)$.

Else $H : \mathbf{M} \to \mathbf{H}$ for some message space $\mathbf{M}$ and hash space $\mathbf{H}$, compose with some map $\phi : \mathbf{H} \to \mathbf{M}$, iterate $W_{i+1} = H(\phi(W_i))$.

Can use parallel version due to van Oorshot and Wiener.

How to use collisions?





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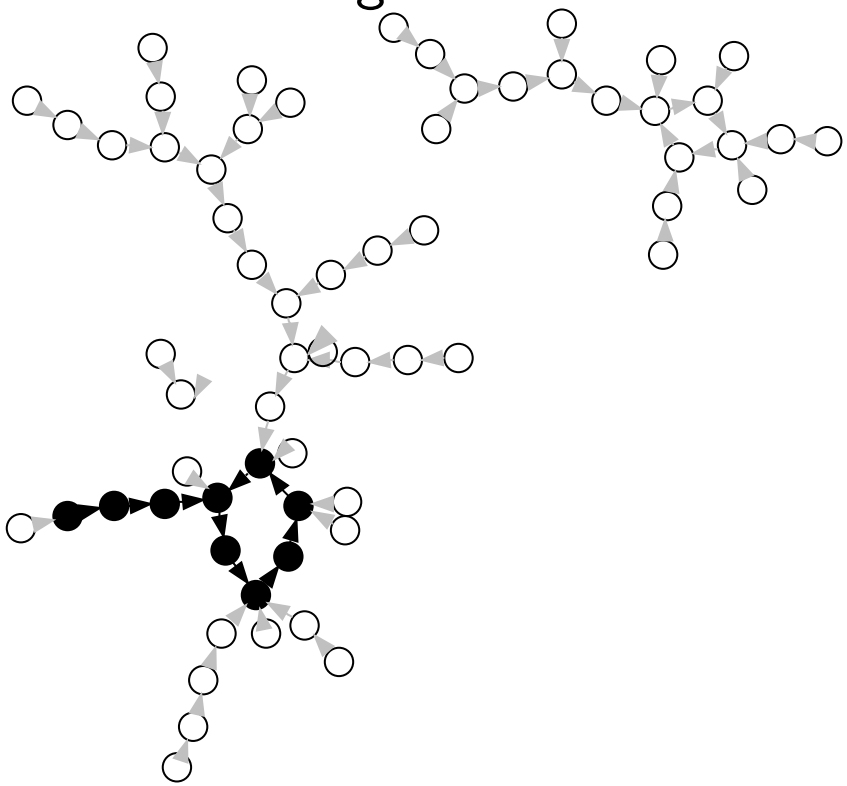

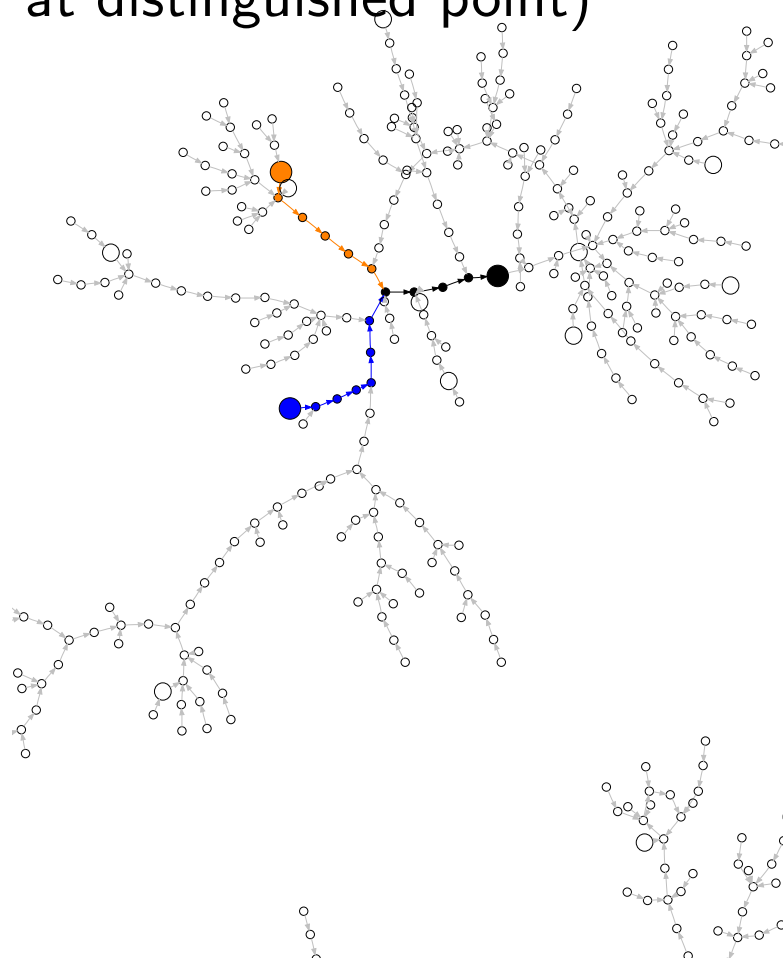

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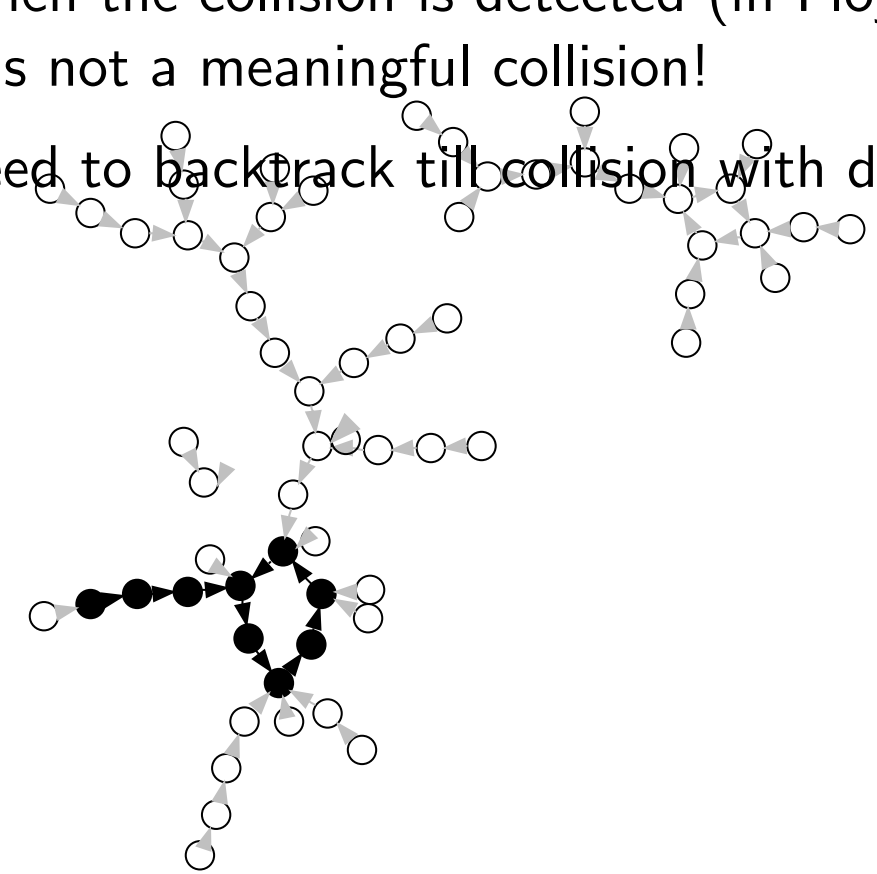

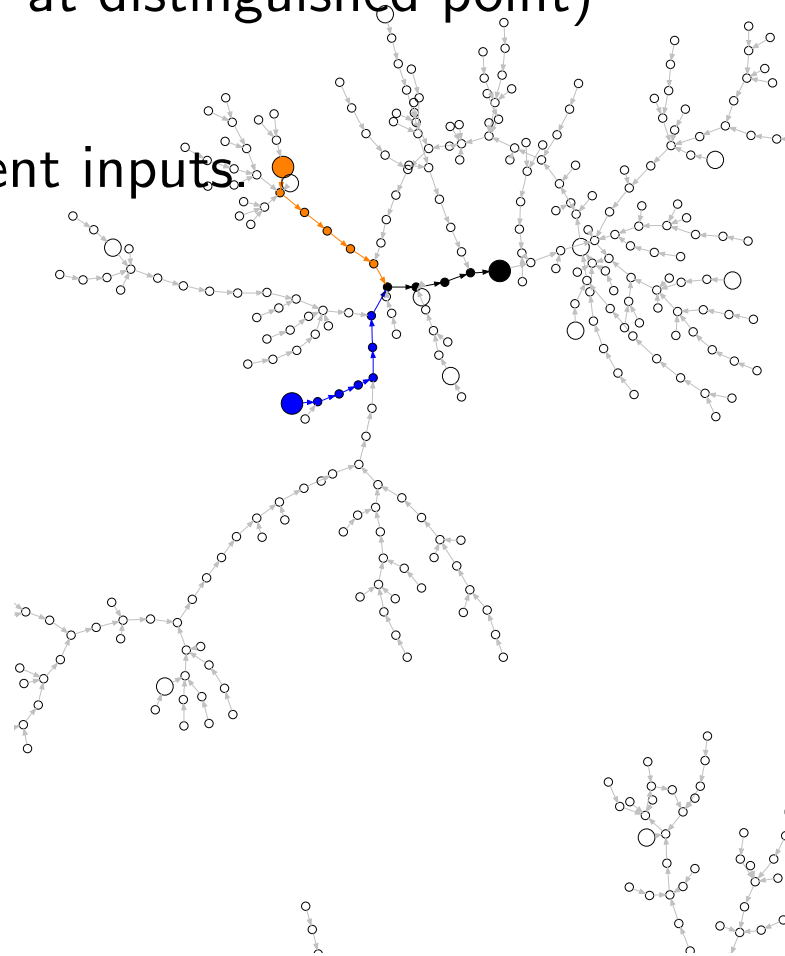

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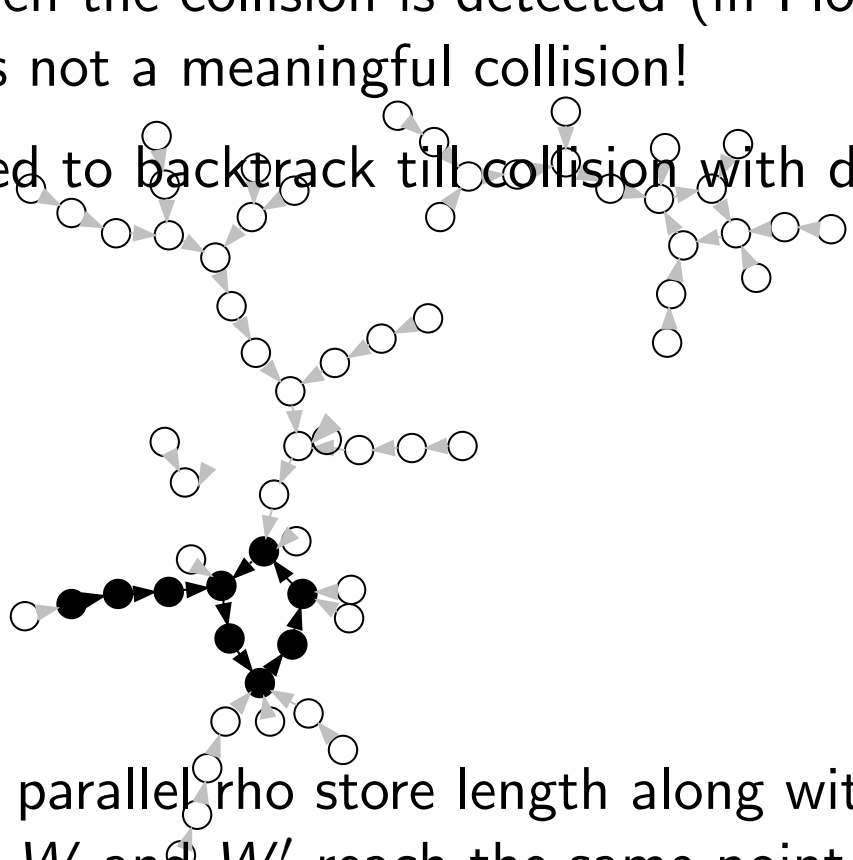

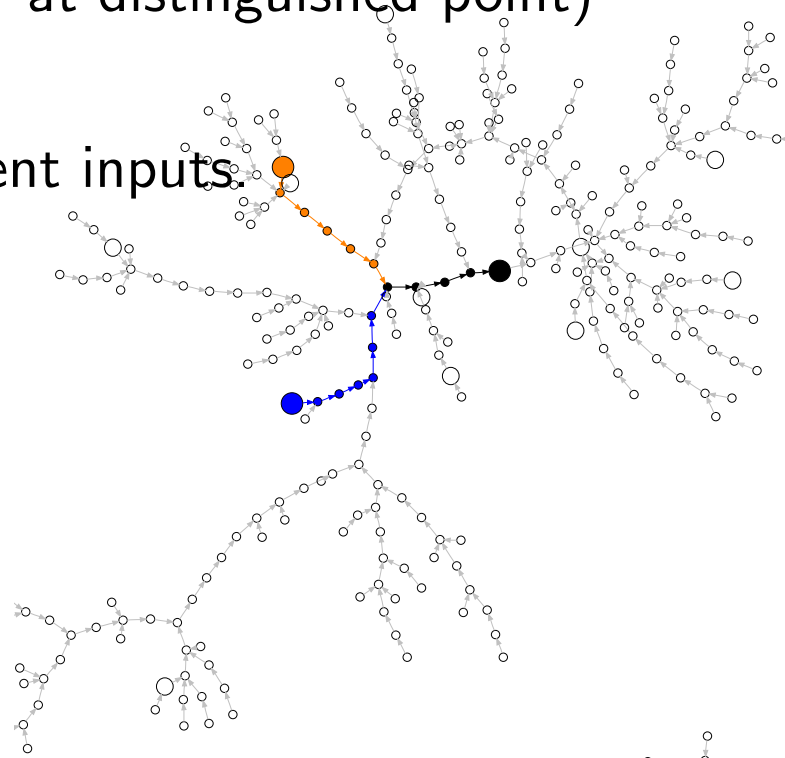

For parallel rho store length along with start and endpoint.
Let W and W' reach the same point after ℓ and ℓ' steps ($\ell < \ell'$)

- Compute $W'' = H^{\ell'-\ell}(W')$.

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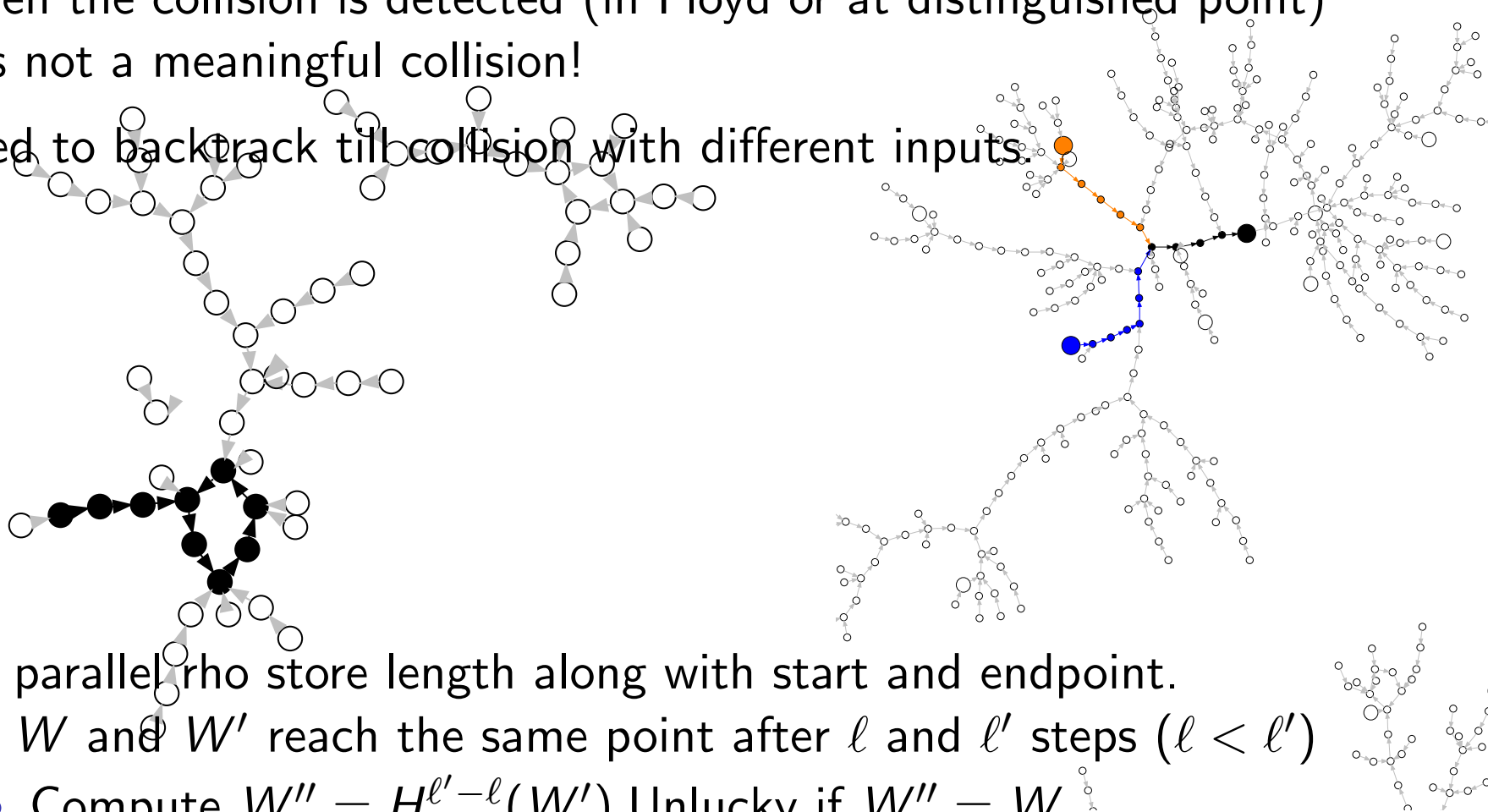

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- Compute $W'' = H^{\ell'-\ell}(W')$.Unlucky if $W'' = W$.
- Iterate the following three steps until a collision is found:
  - Compare W and W''. If they are equal, output W_o and W_o''.
  - Store $W_o = W, W_o'' = W''$.
  - Update $W = H(W), W'' = H(W'')$.

How to get interesting collisions?

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Specify n bit positions in $\mathbf{M}$ that permit variations, make ϕ map to those injectively. Several advanced formats are very flexible in taking macros or invisible images.

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Even more meaningful collisions

- Chosen prefix collision: want $m = p*$ and $m' = p'*$ to collide, where $*$ stands for arbitrary strings.
- Read Predicting the winner of the 2008 US Presidential Elections using a Sony PlayStation 3 for an impressive example of 12 pdf files all having the same hash, using chosen prefix collisions in MD5 and the flexibility of the pdf standard.