Elliptic-curve cryptography VII

Timing attacks and scalar multiplication

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2MMC10 – Cryptology

Timing attacks are not a new phenomenon

Password recovery if server compares letter by letter:

Try `AAA`,

Timing attacks are not a new phenomenon

Password recovery if server compares letter by letter:

Try `AAA`, `BBB`,

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Password recovery if server compares letter by letter:

Try `AAA`, `BBB`, `CCC`, ...

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Password recovery if server compares letter by letter:

Try `AAA`, `BBB`, `CCC`, ..., `CCC` takes slightly longer to fail.
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$\vdots$

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Password is `CRYPTOLOGY`.

1974: Exploit developed by Alan Bell for TENEX operating system.

Reminder: double-and-add method

Compute aP given a and P.

```
a = 44444 # our super secret scalar. No, not that one.
l = a.nbits()
A = a.bits()
R = P
for i in range(l-2,-1,-1):
  R = 2 R
  if A[i] == 1:
    R = R + P
print(R)
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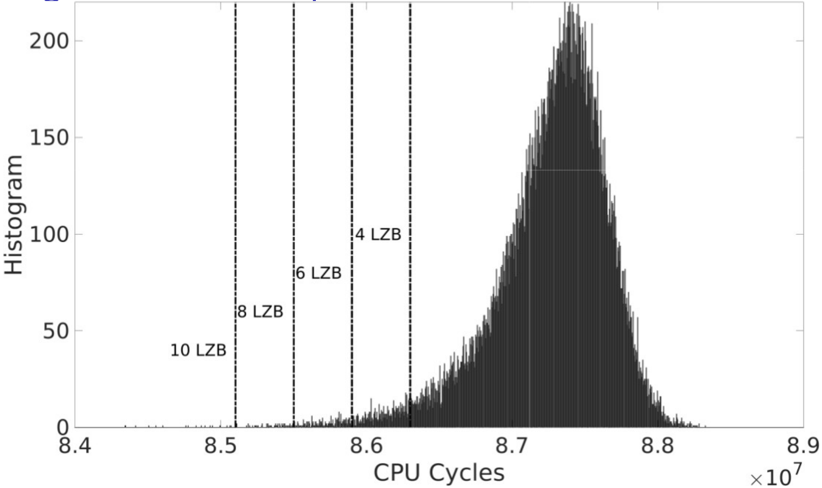
Timings of scalar multiplication on NIST P-256



(Picture from TPM-Fail)

NIST P-256 is an elliptic curve standardized by NIST.
It is a Weierstrass curve modulo a 256-bit prime.

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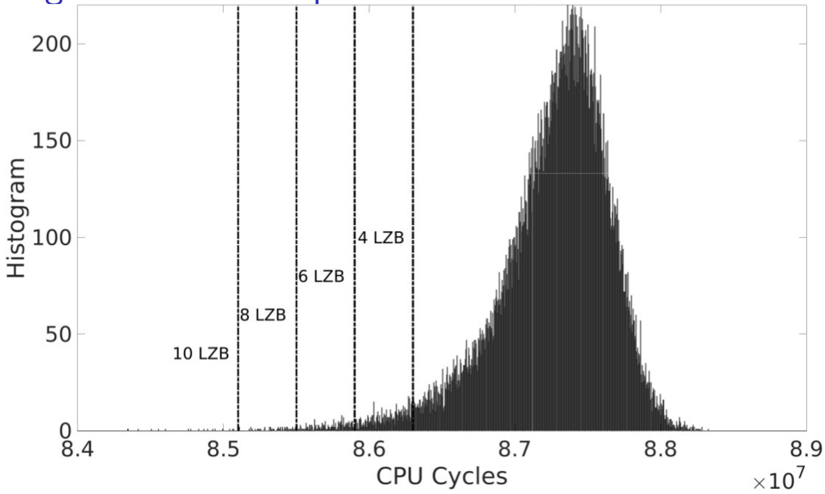

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Timing depends strongly on the length of the scalar, also on Hamming weight.

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  Precompute P, $2P$, and $3P$. Left window is innermost coefficient.

  $$14019P = 4\left(4\left(4\left(4\left(4\left(4\left(3P\right)+P\right)+2P\right)+3P\right)\right)\right)+3P.$$

  Same number of doublings, 4 instead of 7 additions.

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  Start from least-significant bit (coefficient of 2^0)
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  $$14019P = 16\left(16\left(16\left(3P\right)+6P\right)+12P\right)+3P.$$

  Same number of doublings, 3 additions.

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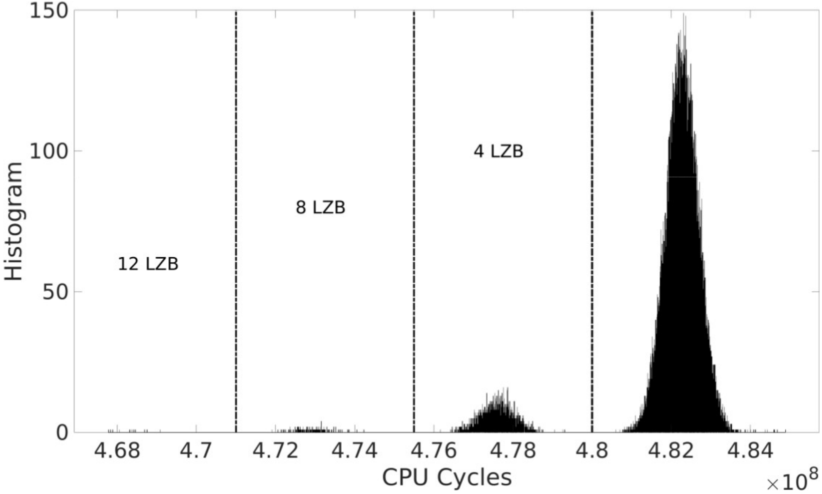

Larger windows reduce the variability through branching but accentuate the length.

(Picture from TPM-Fail)