Introduction to the theory of secret key cryptography

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16 October 2019

Main primitives of secret key / symmetric cryptography

High-level primitives

Low-level primitives

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- Secret key encryption (SKE)
  - Provides: Secrecy
  - Applications: File encryption, communication secrecy

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  - Provides: Integrity & authentication
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  - Provides: Pseudorandom behaviour
  - Applications: Replace random bits / functions with deterministic object

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Low-level primitives

- Pseudorandom generator (PRG) / function (PRF)
  - Provides: Pseudorandom behaviour
  - Applications: Replace random bits / functions with deterministic object
- Cryptographic hash functions
  - Provides: One-wayness, collision resistance
  - Applications: From digital signatures to password hashing and PoW

Secret key encryption

Secret key encryption (SKE)

Secret key cryptography

Definition (Secret key encryption scheme)

A secret key encryption scheme is a tripple of algorithms $\mathcal{E} = (\mathrm{Gen}, \mathrm{Enc}, \mathrm{Dec})$ and a message or plaintext space $\mathcal{M}$ such that the following holds

- Gen is a probabilistic algorithm that outputs a key k. The output space of Gen is called key space $\mathcal{K}$.
- Enc takes as inputs a key $k \in \mathcal{K}$ and message $m \in \mathcal{M}$, and outputs ciphertext $c = \mathrm{Enc}_k(m)$. The output space of Enc is called ciphertext space $\mathcal{C}$.
- Dec is a deterministic algorithm that takes as inputs a key $k \in \mathcal{K}$ and ciphertext $c \in \mathcal{C}$ and outputs a plaintext $m' \in \mathcal{M} : m' = \mathrm{Dec}_k(c)$.

Correctness: $(\forall k \leftarrow \mathrm{Gen}(), \forall m \in \mathcal{M}) : \mathrm{Dec}_k(\mathrm{Enc}_k(m)) = m$

How to define security?

Definition (Perfect secrecy)

A secret key encryption scheme $\mathcal{E} = (\mathrm{Gen}, \mathrm{Enc}, \mathrm{Dec})$ with message space $\mathcal{M}$ is perfectly secret if for every probability distribution over $\mathcal{M}$, every message $m \in \mathcal{M}$, and every ciphertext $c \in \mathcal{C}$ for which $\Pr[C = c] > 0$:

$$\Pr[M = m \mid C = c] = \Pr[M = m]$$

Perfect secrecy considers adversaries $\mathcal{A}$ with $\mathrm{unlimited}$ power.

Is perfect secrecy achievable?

A scheme that is perfectly secret is Vernam's one-time pad (OTP):

Construction (One-time pad)

Let $\mathcal{M} = \{0,1\}^\ell (= \mathcal{K} = \mathcal{C})$, the one-time pad is the encryption scheme consisting of the following three algorithms:

$\mathrm{Gen}()$: *Return* $k \leftarrow_R \{0,1\}^\ell$.

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Main observation behind security proof

For every pair (m, c) of message and ciphertext there exists exactly one key that encrypts m as c.

Is perfect secrecy **efficiently** achievable?

Theorem

Let $\mathcal{E}$ be a perfectly secret encryption scheme over message space $\mathcal{M}$, and let $\mathcal{K}$ be the key space determined by Gen. *Then*

$$|\mathcal{K}| \geq |\mathcal{M}|.$$

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Proof sketch

Assume $|\mathcal{K}| < |\mathcal{M}|$.

1. An arbitrary ciphertext c can only decrypt to $\leq |\mathcal{K}|$ different messages.
2. Consequently, there exist messages m such that $\Pr[M = m \mid C = c] = 0$.
3. If we choose the uniform distribution as message distribution $\Pr[M = m] > 0$.

Hence, $\mathcal{E}$ is not perfectly secure.

Back to square 1: How to define security?

Consider security against efficient (= computationally bounded [= polytime]) adversaries.

Experiment- / game-based security definitions: We define a game that is played by the adversary and analyze its success probability.

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x is sampled from X and $\mathcal{A}$ receives $(\mathrm{Enc}_k(x), h(x))$. $\mathcal{A}$ succeeds if $\mathcal{A}(\mathrm{Enc}_k(x), h(x)) = f(x)$.
($\mathcal{S}$ only receives $h(x)$. $\mathcal{S}$ succeeds if $\mathcal{S}(h(x)) = f(x)$.)

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Definition (Semantic Security (SEM))

A secret key encryption scheme has semantic security if for any efficient adversary $\mathcal{A}$ there exists an efficient simulator $\mathcal{S}$ such that their probabilites of success playing $\mathrm{Exp}^{\mathrm{SEM}}_{\mathcal{E},\mathcal{A}}(n)$ are negligibly close to each other.

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For unbounded adversaries this is equivalent to perfect secrecy.

This definition is cumbersome to work with!

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Definition (Indistinguishable ciphertexts (IND))

A secret key encryption scheme $\mathcal{E}$ has indistinguishable ciphertexts if for all efficient adversaries $\mathcal{A}$ their advantage ε in winning above game is negligible

$$\Pr\left[\mathrm{Exp}^{\mathrm{IND}}_{\mathcal{E},\mathcal{A}}(n) = 1\right] = \frac{1}{2} + \varepsilon.$$

This definition is a lot easier to work with and equivalent to SEM!

Is IND efficiently achievable?

We first need tooling.

Definition (Pseudorandom generator (PRG))

Let ℓ be a polynomial and let G be a deterministic, efficient algorithm that implements a function $G : \{0,1\}^n \to \{0,1\}^{\ell(n)}$. We say G is a secure PRG if the following two conditions hold:

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PRG's exist if one-way functions exist. Will see examples later.

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Construction (PRG-ENC)

Let $n \in \mathbb{N}$ be the security parameter, let $\mathcal{M} = \{0,1\}^{\ell(n)} (= \mathcal{C})$, and let G be a PRG as defined above. The PRG-ENC encryption scheme consists of the following three algorithms:

$\mathrm{Gen}(1^n)$: *Return* $k \leftarrow_R \{0,1\}^n$.

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PRG-ENC is IND secure

Proof by reduction. If there exists $\mathcal{A}$ that can distinguish ciphertexts of PRG-ENC in time t with advantage ε then the following algorithm $\mathcal{D}$ runs in time $\approx t$ and succeeds in distinguishing G with advantage $\varepsilon' = \varepsilon$.

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PRG-ENC is IND secure

Theorem

If there exists $\mathcal{A}$ that can distinguish ciphertexts of PRG-ENC in time t with advantage ε then the algorithm $\mathcal{D}$ from above runs in time $\approx t$ and succeeds in breaking G with advantage $\varepsilon' = \varepsilon$.
Hence, if G is a secure PRG, then PRG-ENC has indistinguishable ciphertexts.

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- SEM, IND, and perfect secrecy define $\mathcal{A}$'s goal
- What about $\mathcal{A}$'s attack capabilities?
- In this sense they are unrealistic single message notions.

Is this realistic?

Or rather this.

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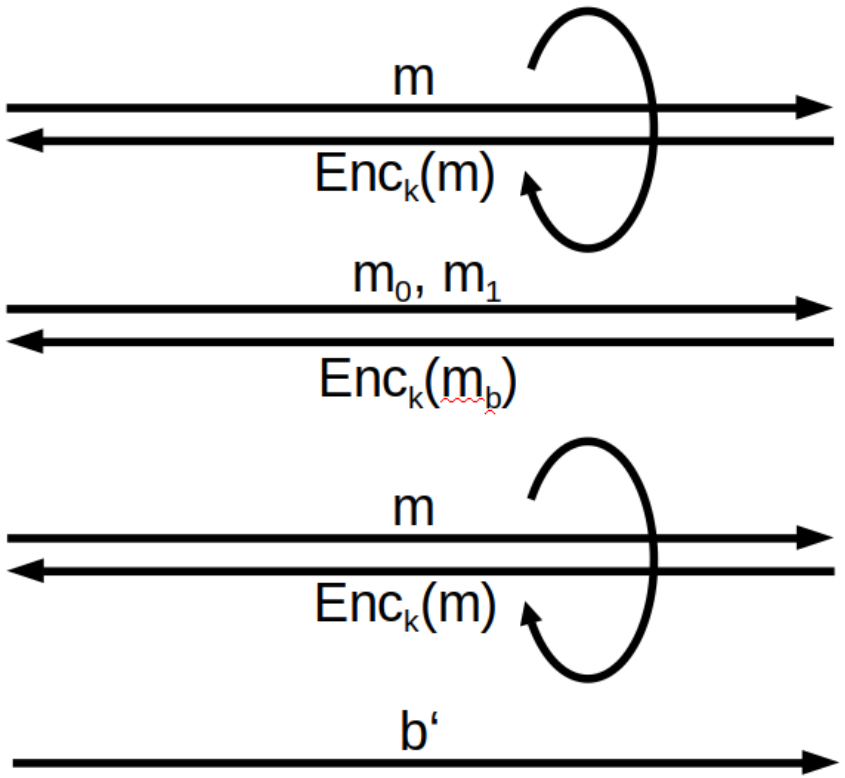
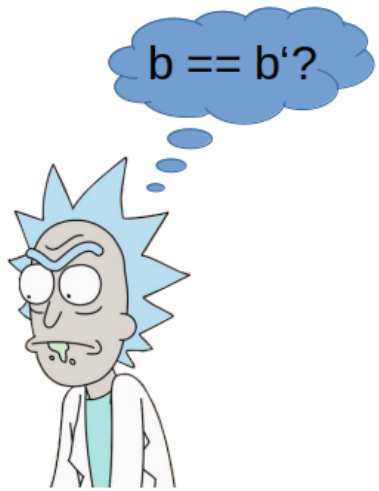
- Want to model the worst case: Let $\mathcal{A}$ choose messages that get encrypted!

IND under chosen plaintext attacks (IND-CPA)





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Definition (IND-CPA)

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Note: This definition is equivalent to SEM-CPA.

IND-CPA secure SKE

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Proof idea.

Send m_0 to $\mathrm{Enc}_k(\cdot)$ and compare result with challenge ciphertext. □

Pseudorandom function families

A keyed function is a two input function $\mathsf{F} : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$ where the first input is called the key and denoted k. We will write $\mathsf{F}_k(x) \stackrel{def}{=} \mathsf{F}(k, x)$.

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Definition (Pseudorandom function family (PRF))

Let $\mathsf{F} : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be an efficient, length-preserving, keyed function. We say F is a pseudorandom function if for all efficient distinguishers $\mathcal{D}$ the distinguishing advantage ε is negligible, where

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PRF's exist if PRG's exist [GGM'84]. For length doubling PRG G define

$$\mathsf{F}_k(x) \stackrel{def}{=} \mathsf{G}\left(\ldots \mathsf{G}\left(\mathsf{G}(k)_{x_1} \right)_{x_2} \ldots \right)_{x_n} .$$

Pseudorandom permutation families

Formal model for block ciphers is PRP.

Definition (Pseudorandom permutation family (PRP))

Let $n \in \mathbb{N}$ be the security parameter,
$F : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be an efficient, length-preserving, keyed permutation. We say F is a family of pseudorandom permutations (PRP) if for all efficient distinguishers $\mathcal{D}$ the distinguishing advantage ε is negligible, where

$$\varepsilon = \left| \Pr_{k \leftarrow_R \{0,1\}^n} \left[\mathcal{D}^{F_k(\cdot), F_k^{-1}(\cdot)}(1^n) = 1 \right] - \Pr_{f_n \leftarrow_R \mathrm{PERM}_n} \left[\mathcal{D}^{f_n(\cdot), f_n^{-1}(\cdot)}(1^n) = 1 \right] \right|,$$

where PERM_n denotes the set of all permutations over $\{0,1\}^n$.

A PRP is a PRF (Switching-Lemma) but not vice-versa.

IND-CPA-secure SKE

Construction (PRF-ENC)

Let $n \in \mathbb{N}$ be the security parameter, let $\mathcal{M} = \{0,1\}^n (= \mathcal{C} = \mathcal{K})$, and let F be a length-preserving PRF as defined above. The PRF-ENC encryption scheme consists of the following three algorithms:

$\mathrm{Gen}(1^n)$: *Return $k \leftarrow_R \{0,1\}^n$.*

$\mathrm{Enc}_k(m)$: *Sample $r \leftarrow_R \{0,1\}^n$, compute $\bar{c} = m \oplus \mathsf{F}_k(r)$, and return $c = \langle r, \bar{c} \rangle$.*

$\mathrm{Dec}_k(c)$: *Parse c as $\langle r, \bar{c} \rangle$. Return $m' = \bar{c} \oplus \mathsf{F}_k(r)$.*

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Correctness

$$\mathrm{Dec}_k(\mathrm{Enc}_k(m)) = (m \oplus \mathsf{F}_k(r)) \oplus \mathsf{F}_k(r) = m$$

PRF-ENC is IND-CPA secure

Proof idea. Similar to PRG-ENC. Given $\mathcal{A}$ that breaks IND-CPA of PRF-ENC in time t, with advantage ε then the following algorithm $\mathcal{D}$ runs in time $\approx t$ and succeeds in distinguishing F with advantage $\varepsilon' \approx \varepsilon$.

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Construction (Distinguisher $\mathcal{D}$)

Given access to oracle $\mathcal{O} : \{0,1\}^n \to \{0,1\}^n$*:*

1. *Run* $m_0, m_1 \leftarrow \mathcal{A}^{\mathrm{Enc}'(\cdot)}(1^n)$
2. *Set* $b \leftarrow_R \{0,1\}, r^* \leftarrow_R \{0,1\}^n, \bar{c}^* = m_b \oplus \mathcal{O}(r^*)$
3. *Run* $b' \leftarrow \mathcal{A}^{\mathrm{Enc}'(\cdot)}(\langle r^*, \bar{c}^* \rangle)$
4. *Return 1 if* $b = b'$*, otherwise 0*

where $\mathrm{Enc}'(\cdot)$ *computes* $r \leftarrow_R \{0,1\}^n, \bar{c} = m_b \oplus \mathcal{O}(r)$ *and returns* $\langle r, \bar{c} \rangle$.

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$$= \left| \Pr \left[\mathsf{Exp}^{\mathsf{IND-CPA}}_{\mathrm{PRF-ENC},\mathcal{A}}(n) = 1 \right] - \Pr \left[\mathsf{Exp}^{\mathsf{IND-CPA}}_{\widetilde{\mathrm{PRF-ENC}},\mathcal{A}}(n) = 1 \right] \right|$$

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$$
\begin{aligned}
\varepsilon' &= \left| \Pr_{k \leftarrow_R \{0,1\}^n} \left[\mathcal{D}^{\mathsf{F}_k(\cdot)}(1^n) = 1 \right] - \Pr_{f_n \leftarrow_R \mathsf{FUNC}_n} \left[\mathcal{D}^{f_n(\cdot)}(1^n) = 1 \right] \right| \\
&= \left| \Pr \left[\mathrm{Exp}^{\mathsf{IND-CPA}}_{\mathrm{PRF-ENC},\mathcal{A}}(n) = 1 \right] - \Pr \left[\mathrm{Exp}^{\mathsf{IND-CPA}}_{\widetilde{\mathrm{PRF-ENC}},\mathcal{A}}(n) = 1 \right] \right| \\
&= \left| \frac{1}{2} + \varepsilon - \left(\frac{1}{2} + \frac{q}{2^n} \right) \right| = \left| \varepsilon - \frac{q}{2^n} \right|
\end{aligned}
$$

PRF-ENC is IND-CPA secure

Theorem

If there exists $\mathcal{A}$ that can distinguish ciphertexts of PRF-ENC during a CPA-experiment in time t with advantage ε then the algorithm $\mathcal{D}$ from above runs in time $\approx t$ and succeeds in breaking F *with advantage $\varepsilon' \geq \varepsilon - q/2^n$.*
Hence, if F *is a secure PRF, then PRF-ENC has indistinguishable ciphertexts under chosen plaintext attacks.*

Arbitrary length messages

PRF-ENC only works for n-bit messages.

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For ℓn-bit message $m = (m_1 \| m_2 \| \ldots \| m_\ell)$ ciphertext is

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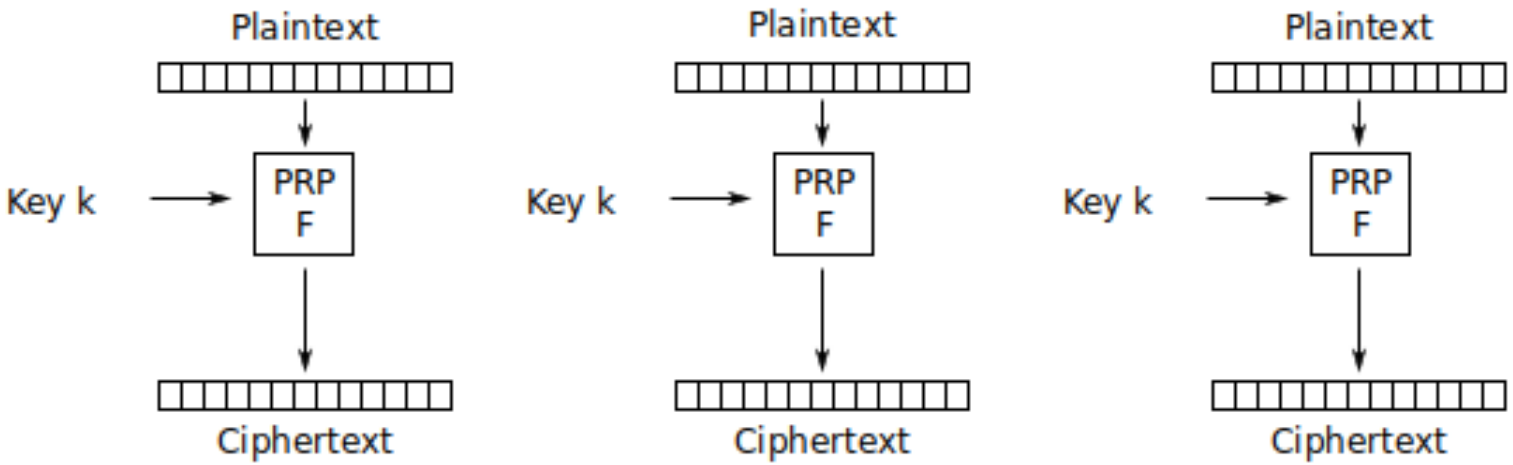
Pretty inefficient!

Solution: Modes of operation

Electronic code book mode (ECB)



Electronic Codebook (ECB) mode encryption

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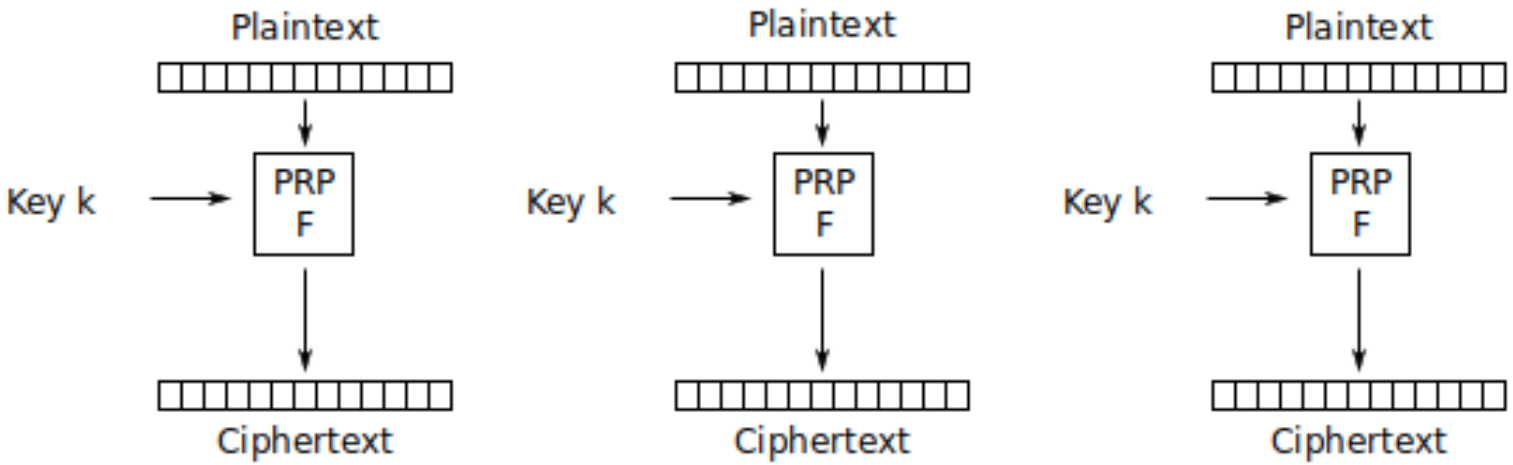
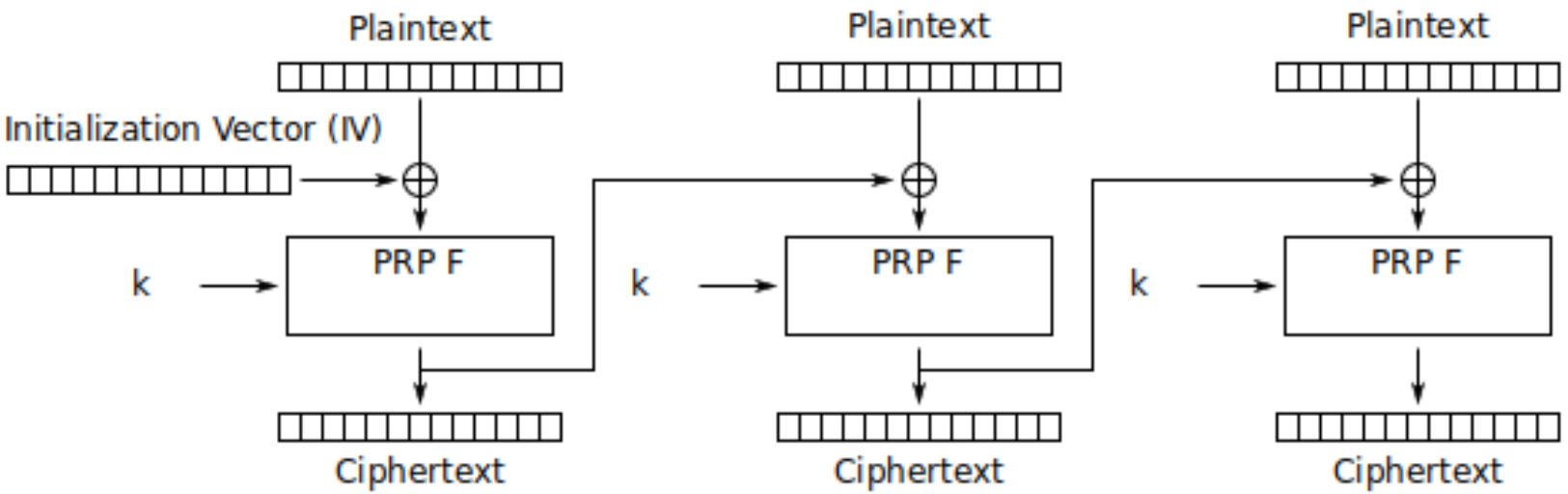


Electronic Codebook (ECB) mode encryption

Deterministic! Even worse, not even IND for single message attacks!

(Consider $m_0 = m\|m; m_1 = m\|m'$ for $m, m' \in \{0,1\}^n$)

Cipher block chaining mode (CBC)



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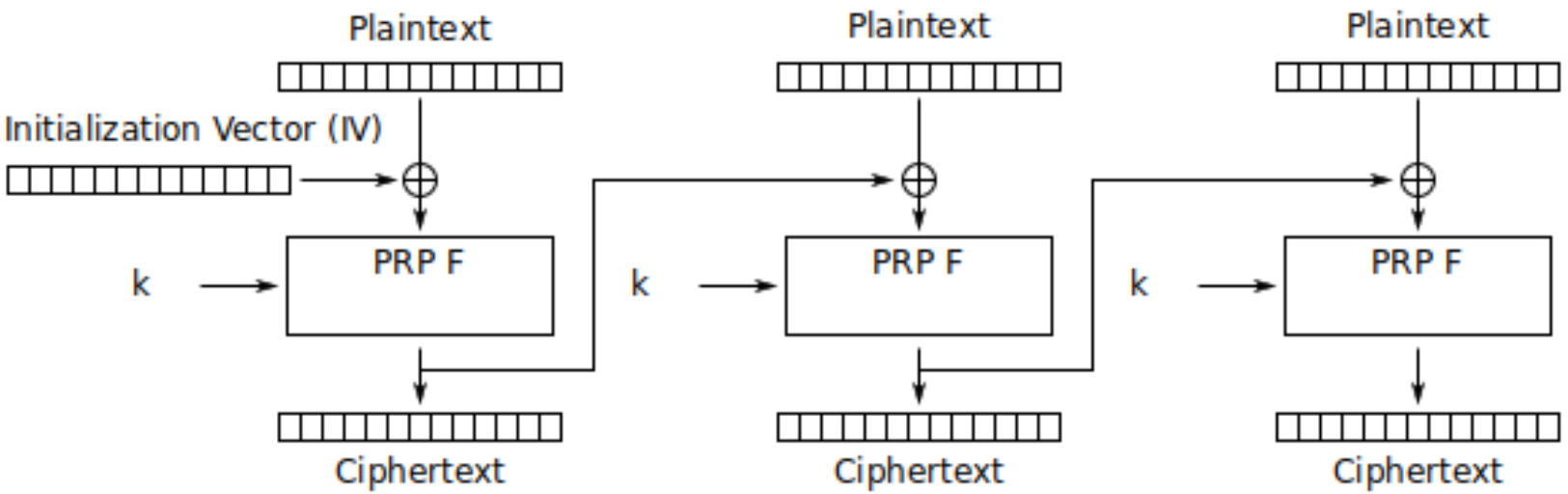
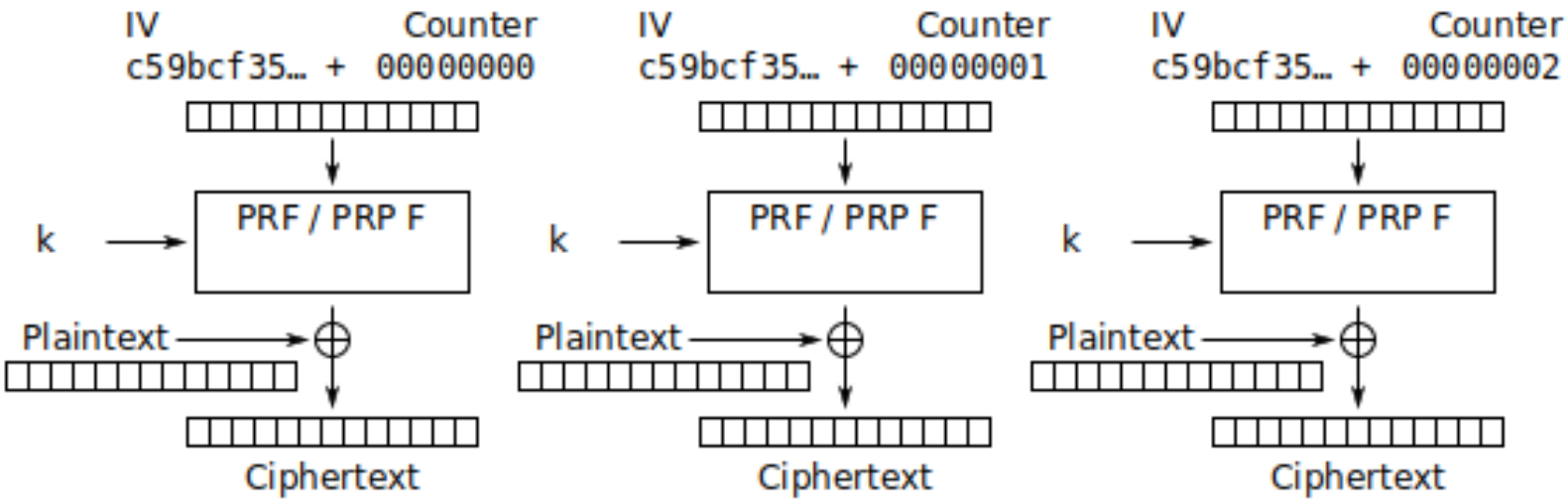


Cipher Block Chaining (CBC) mode encryption

IND-CPA if F is a PRP. IV has to be random, if it is predictable CBC is vulnerable!

Counter mode (CTR)



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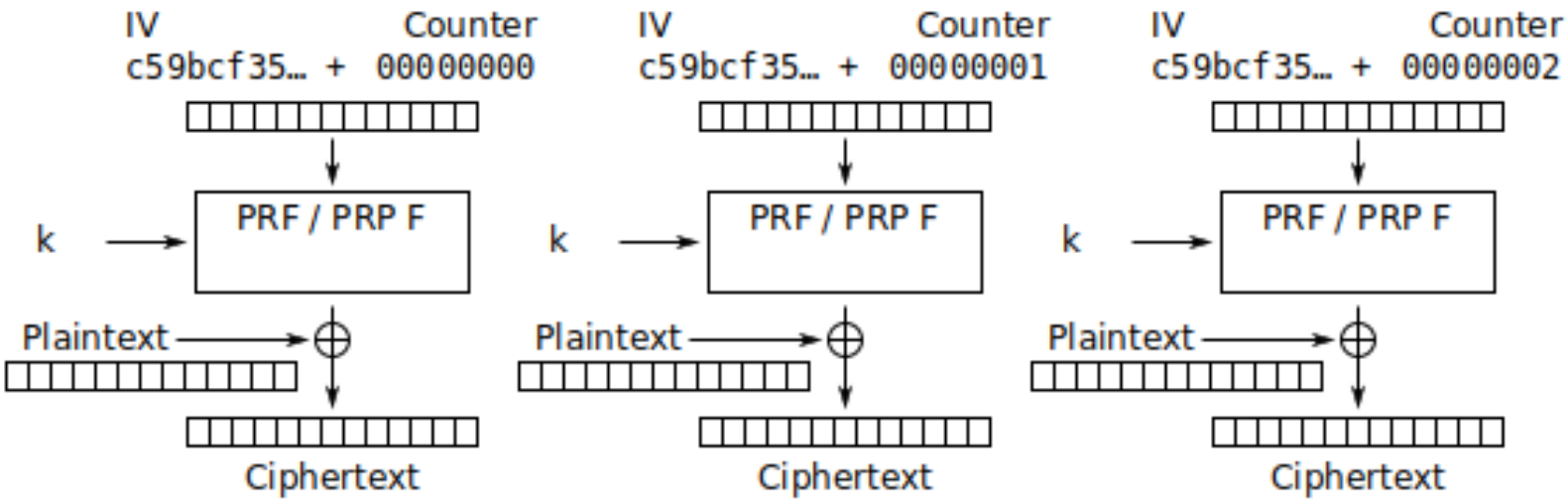

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- Want to model the worst case: Let $\mathcal{A}$ choose ciphertexts that get decrypted!

IND under chosen ciphertext attacks

$\mathrm{Exp}^{\mathrm{IND-CCA}}_{\mathcal{E},\mathcal{A}}(n)$:

1. $k \leftarrow \mathrm{Gen}(1^n)$
2. $m_0, m_1 \leftarrow \mathcal{A}^{\mathrm{Enc}_k(\cdot),\mathrm{Dec}_k(\cdot)}(1^n)$ with $m_0, m_1 \in \mathcal{M} \wedge |m_0 = m_1|$
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Definition (IND-CCA)

A secret key encryption scheme $\mathcal{E}$ has indistinguishable ciphertexts under chosen ciphertext attacks if for all efficient adversaries $\mathcal{A}$ their advantage ε in winning above game is negligible

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This definition is equivalent to SEM-CCA.

MAC

Message authentication

- Sometimes we want more than secrecy!
- Acknowledgement of receipt, social communication, source of executable, ...

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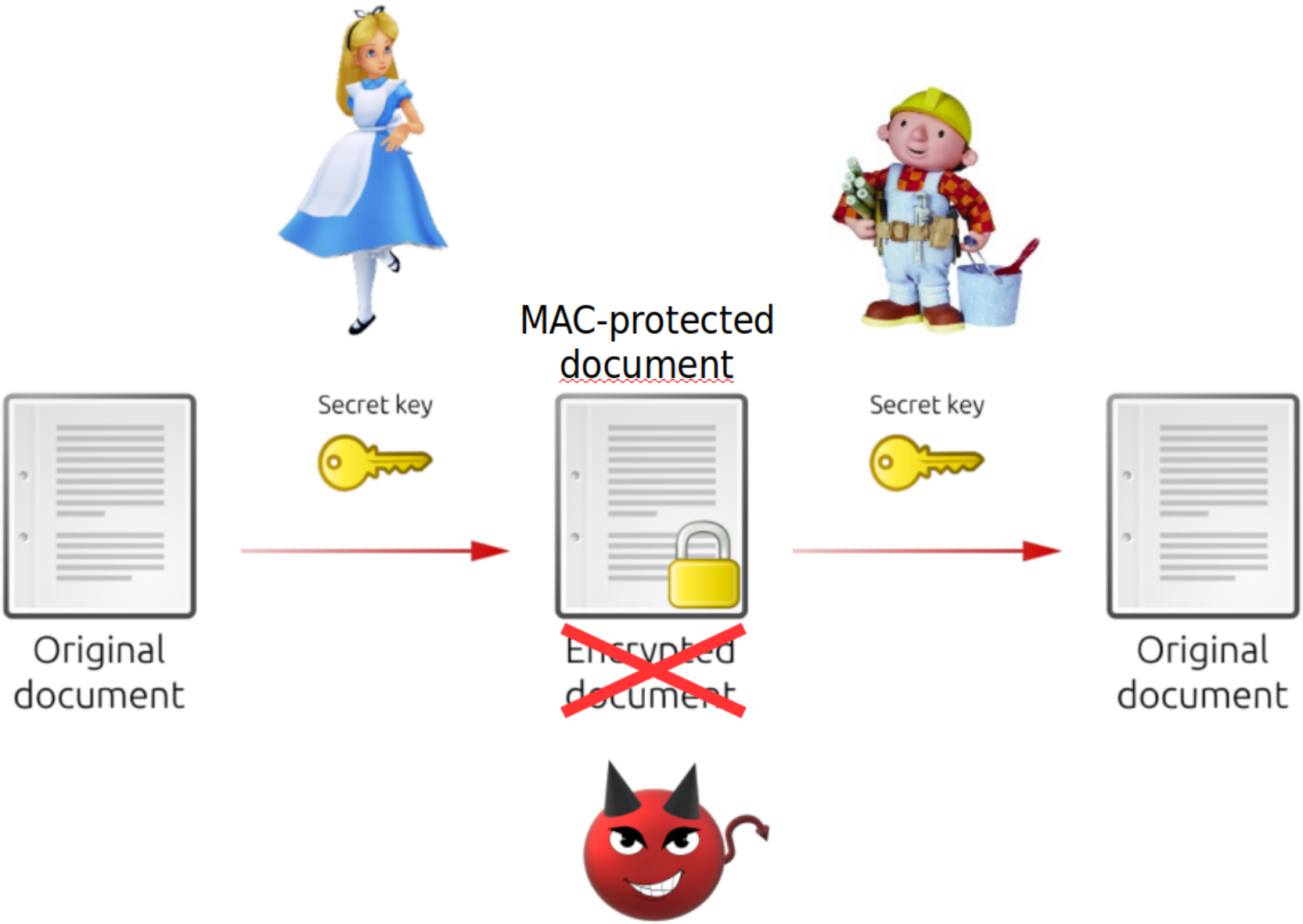
- Encryption $\stackrel{?}{\Rightarrow}$ Authenticity / integrity?
  - PRG-ENC, PRF-ENC, ... any stream cipher allows controlled bit-flips. If format is known this may be disastrous
  - Block ciphers make similar attacks harder but no guarantees.
  - ECB-mode allows to switch order of blocks, repeat blocks, etc.

MAC


MAC-protected document
Secret key
Secret key
Original document
Original document

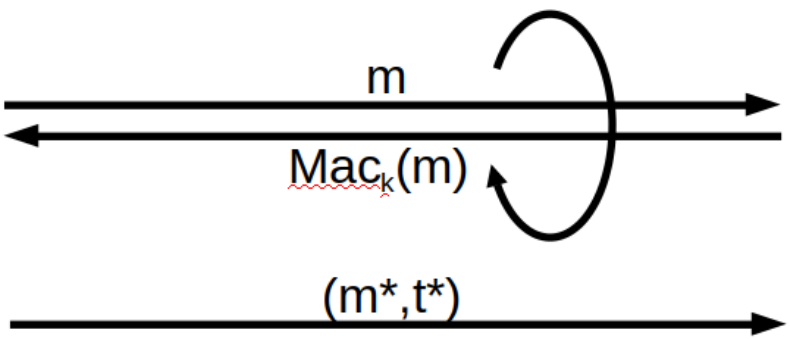
Message authentication codes (MAC)

Definition (message authentication code)

A message authentication code or MAC is a tuple of probabilistic polynomial-time algorithms $\text{MAC} = (\text{Gen}, \text{Mac}, \text{Vrfy})$ over a message space $\mathcal{M}$, fulfilling the following:

- Gen is a probabilistic algorithm that on input 1^n outputs a key k. The output space of Gen is called the key space $\mathcal{K}$.
- Mac takes as input a key $k \in \mathcal{K}$ and a message $m \in \mathcal{M}$, and outputs a tag $t \in \mathcal{T}$. The output space of Mac is called tag space $\mathcal{T}$.
- Vrfy is a deterministic algorithm that takes as inputs a key $k \in \mathcal{K}$, a message $m \in \mathcal{M}$, and a tag $t \in \mathcal{T}$, and outputs a bit $b \in \{0, 1\}$.
- Correctness: For every n, every $k \leftarrow \text{Gen}(1^n)$, and every $m \in \mathcal{M}$ it holds that $\text{Vrfy}_k(m, \text{Mac}_k(m)) = 1$.

Existential unforgeability under (adaptive) chosen message attacks (EU-CMA)



Existential unforgeability under (adaptive) chosen message attacks (EU-CMA)

$\mathsf{Exp}^{\mathrm{EU-CMA}}_{\mathsf{MAC},\mathcal{A}}(n)$

1. $k \leftarrow \mathrm{Gen}(1^n)$
2. $(m, t) \leftarrow \mathcal{A}^{\mathrm{MAC}_k(\cdot)}(1^n)$. Let $\{m_i\}_1^q$ denote $\mathcal{A}$'s queries to MAC_k
3. If $\mathrm{VRFY}_k(m, t) := 1$ and $m \notin \{m_i\}_1^q$ return 1
4. Else return 0.

Existential unforgeability under (adaptive) chosen message attacks (EU-CMA)

Definition (EU-CMA)

A message authentication code $\mathsf{MAC} = (\mathrm{Gen}, \mathrm{Mac}, \mathrm{Vrfy})$ over a message space $\mathcal{M}$ is existentially unforgeable under an adaptive chosen-message attack, or just secure, if for all efficient adversaries $\mathcal{A}$ the success probability ε in winning $\mathsf{Exp}^{\mathrm{EU-CMA}}_{\mathsf{MAC},\mathcal{A}}(n)$ is negligible, where

$$\varepsilon = \Pr\left[\mathsf{Exp}^{\mathrm{EU-CMA}}_{\mathsf{MAC},\mathcal{A}}(n) = 1\right]$$

Remarks

- There exists a constant time attack with success probability $1/|\mathcal{T}|$ against every MAC $\Rightarrow$ Tags must not be too short

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- There exists a constant time attack with success probability $1/|\mathcal{T}|$ against every MAC $\Rightarrow$ Tags must not be too short
- MAC's do not prevent replay attacks!
- Replay attacks have to be handled on protocol level (e.g., using sequence numbers).

PRF is a MAC

Theorem

A secure PRF F *leads a secure MAC with*

$\mathrm{Gen}(1^n)$ *returns* $k \leftarrow_R \{0,1\}^n$.

$\mathrm{MAC}_k(m)$ *returns* $t = \mathsf{F}_k(m)$.

$\mathrm{VRFY}_k(m,t)$ *returns 1 if* $t = \mathsf{F}_k(m)$*, and 0 otherwise.*

Proof idea

Build distinguisher that simulates experiment using its oracle instead of F. A valid forgery must be on a new message. So if oracle is random, tag is a correct guess for a random function at some point m that was not queried. If $\mathcal{A}$ succeeds more often when the oracle was F, this allows to distinguish F as for PRF-ENC.

CBC-MAC

Construction

Let F *be an efficient, length-preserving keyed function over* $\{0,1\}^n$. *CBC-MAC has message space* $\mathcal{M} = (\{0,1\}^{\ell n})$. *The algorithms are as follows:*

$\mathrm{Gen}(1^n)$ *returns* $k \leftarrow_R \{0,1\}^n$.

$\mathrm{MAC}_k(m)$ *upon input key* $k \in \{0,1\}^n$ *and a message* m *of length* ℓn, *do the following:*

1. *Denote* $m = m_1, \ldots, m_\ell$ *where each* m_i *is of length* n, *and set* $t_0 = 0^n$.
2. *For* $i = 1$ *to* ℓ, *set* $t_i \leftarrow \mathsf{F}_k(t_{i-1} \oplus m_i)$.
3. *Output* t_ℓ.

$\mathrm{VRFY}_k(m,t)$ *returns 1 if* $t = \mathrm{MAC}_k(m)$, *and 0 otherwise.*

Variable message length CBC-MAC

- CBC-MAC is not secure for variable length messages

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Solutions for variable ℓ:

- Derived key: Compute $k' = \mathsf{F}_k(\ell)$ and use k' to compute $t = \mathrm{MAC}_{k'}(m)$
- Prepend length: Compute $t = \mathrm{MAC}_k(\ell \| m)$.
- Encrypted tag: Use two keys $k_1, k_2 \in \{0,1\}^n$, compute $t' = \mathrm{MAC}_{k_1}(m)$ and output $t = \mathsf{F}_{k_2}(t')$. We can generate k_1, k_2 from a single key using F as a length-doubling PRG ($< k_1, k_2 > = < \mathsf{F}_k(0), \mathsf{F}_k(1) >$)

Padding

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Solution: Padding

- Expand message to match multiple of block length.
- Usually injective function $\mathrm{Pad} : \{0,1\}^* \to (\{0,1\}^n)^*$.
- E.g., $m \to m \| 10^*$.
- Properties depend on cryptographic application:
  - Encryption - invertible
  - MAC - injective
- Often used for additional purposes: Randomization, or encoding message length.

Secrecy + Authenticity

- We actually want IND-CCA and EU-CMA security of our connections.

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- MAC-then-Encrypt. $t = \mathrm{MAC}_{k_2}(m),\ c = \mathrm{Enc}_{k_1}(m \| t)$.
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- Encrypt-then-MAC. $c = \mathrm{Enc}_{k_1}(m),\ t = \mathrm{MAC}_{k_2}(c)$.
  Secure! (And the generic way to turn an IND-CPA secure encryption into an IND-CCA secure one.)

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Options:

- Encrypt-and-MAC: $c = \mathrm{Enc}_{k_1}(m),\ t = \mathrm{MAC}_{k_2}(m)$.
  Possibly insecure as MAC might leak!
- MAC-then-Encrypt. $t = \mathrm{MAC}_{k_2}(m),\ c = \mathrm{Enc}_{k_1}(m \| t)$.
  Possibly insecure but counter-examples are more involved
- Encrypt-then-MAC. $c = \mathrm{Enc}_{k_1}(m),\ t = \mathrm{MAC}_{k_2}(c)$.
  Secure! (And the generic way to turn an IND-CPA secure encryption into an IND-CCA secure one.)
- Or the most simple one: Use Authenticated encryption (AE)!

Conclusion

- We covered secret key encryption schemes and their security.
- We covered message authentication codes and their security.
- On the way we looked at PRFs and PRGs.

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Thank you!
Questions?