

# Block ciphers and AES

Cryptology, 2016 Autumn

Joan Daemen

Institute for Computing and Information Sciences

Radboud University

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# Outline

Introduction

Block cipher model and security definition

Data Encryption Standard (DES)

Advanced Encryption Standard (AES)

Encryption modes of block ciphers

Authentication modes of block ciphers



# Currently we are here...

## Introduction

Block cipher model and security definition

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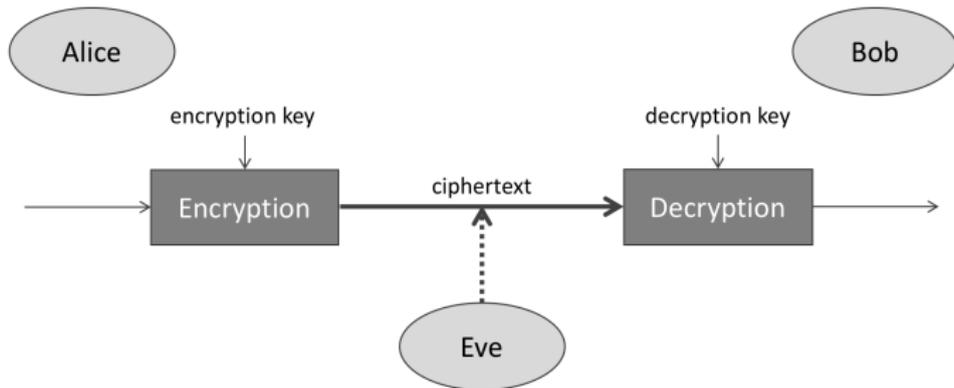
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# Encryption



- ▶ Alice: sender, enciphers message to cryptogram using key
- ▶ Bob: receiver, deciphers cryptogram to message using key
- ▶ Eve: eavesdropper, does not have key



## The one-time pad

$$\begin{array}{r} \text{message} = \\ \text{keystream} = \\ \hline \text{cryptogram} = \end{array} \begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ \hline 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ \hline \end{array} \oplus$$



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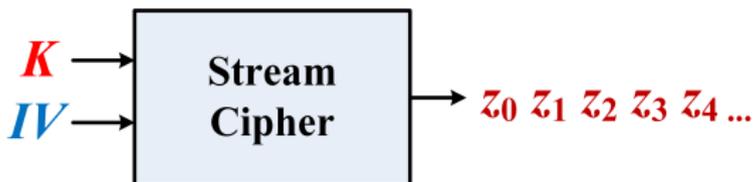
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Provably secure if keystream is *fully random*



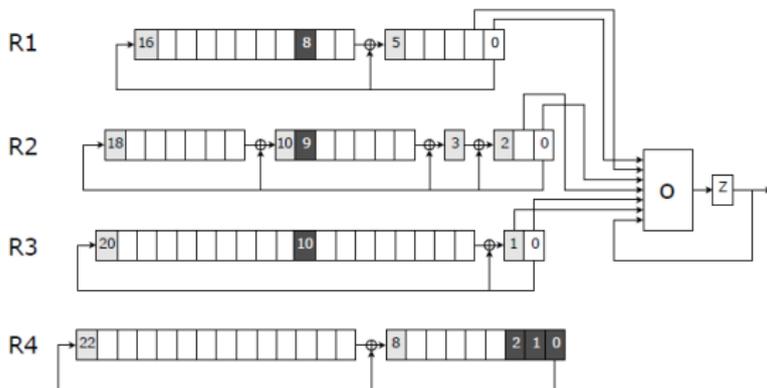
## Stream cipher



- ▶ Generates keystream bits  $z_t$  from
  - $K$ : secret, typically 128 or 256 bits
  - $IV$ : initial value, for generating multiple keystreams per key
- ▶  $z_t$  can be a bit or a sequences of bits, e.g. a 32-bit word



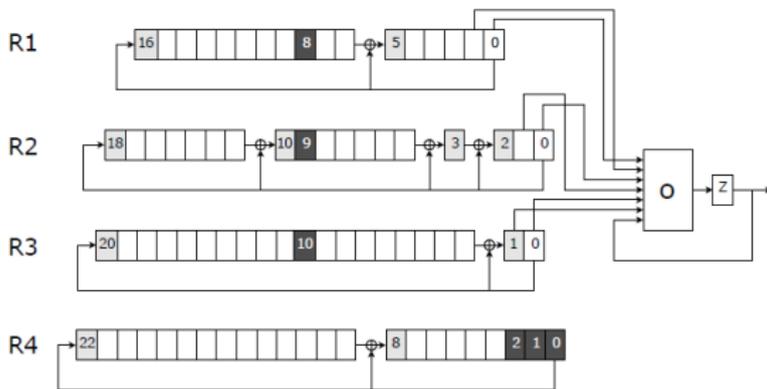
## Example: DECT Stream Cipher



- ▶ In use in hundreds of millions of wireless phones
- ▶ 4 LFSRs with coprime lengths: large period
- ▶ top 3 clocked 2 or 3 times in between time steps  $t$



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- ▶ practically broken with statistical key recovery attack



## Example: RC4 [Ron Rivest] stream cipher

- ▶ State is array of 256 bytes
- ▶ Simple and elegant update function and output function
- ▶ Software-oriented

```
i := 0
j := 0
while GeneratingOutput:
  i := (i + 1) mod 256
  j := (j + S[i]) mod 256
  swap values of S[i] and S[j]
  K := S[(S[i] + S[j]) mod 256]
  output K
endwhile
```

- ▶ Used in TLS and WEP
- ▶ Biases in keystream
- ▶ Practically broken in several use cases



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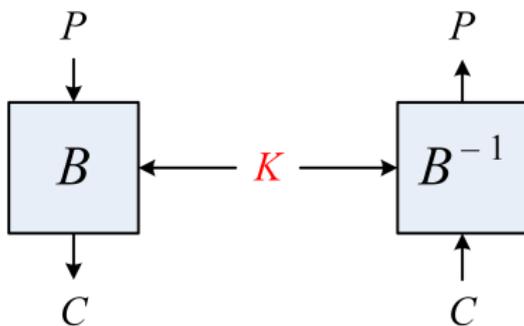
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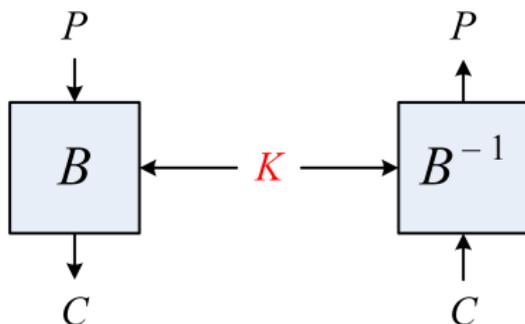
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- ▶ Permutation  $B$  operating on  $\mathbb{Z}_2^b$  with  $b$  the block length



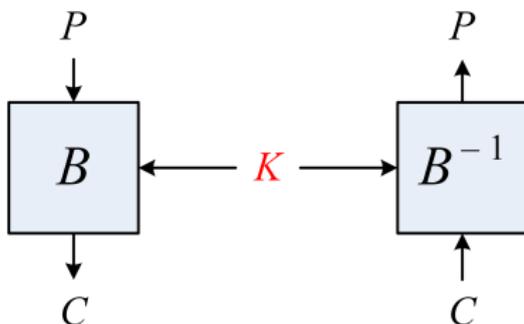
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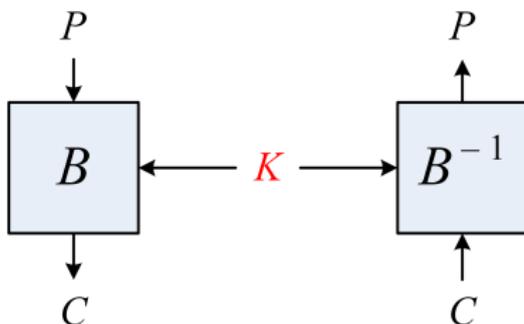
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  - efficient knowing the secret key  $K$
  - infeasible otherwise



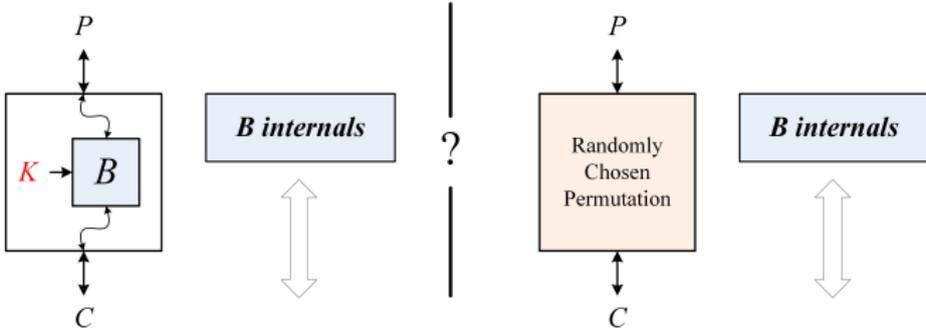
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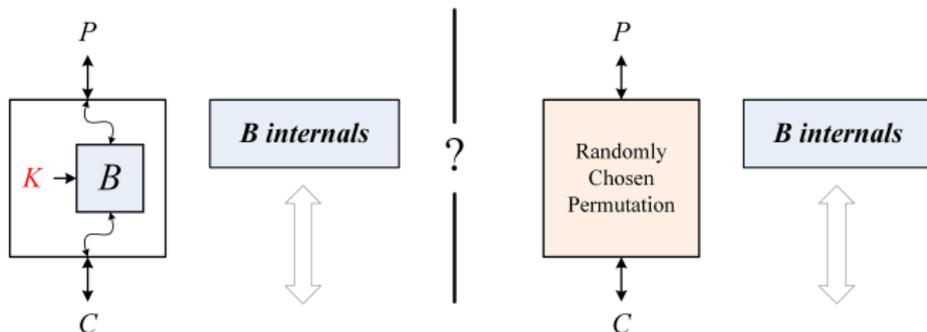
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- ▶ Dimensions: block length  $b$  and **key length**



# Pseudorandom Permutation (PRP) security



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- ▶ Infeasibility to distinguish  $B[K]$  from **random permutation**
- ▶ Distinguishing should have expected effort that is out of reach

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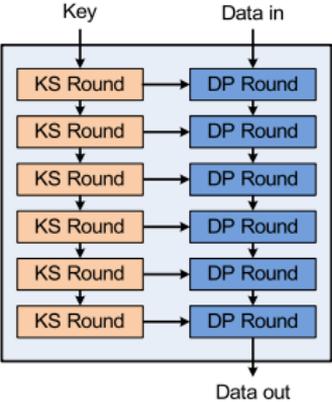
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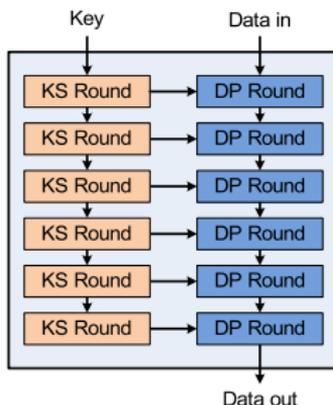
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# Iterative block ciphers



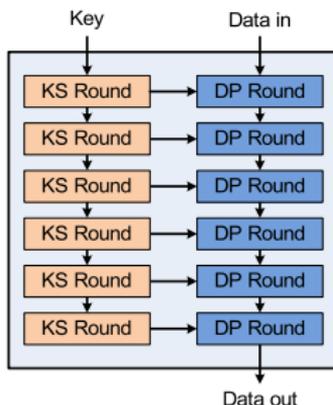
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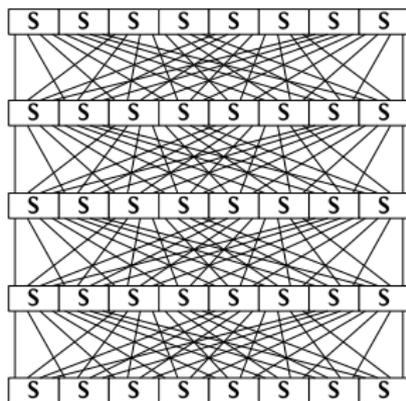
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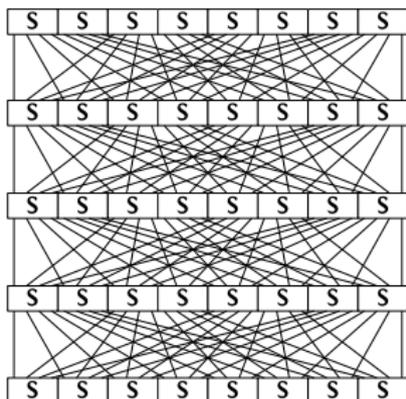
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  - ... that depends on a round key
- ▶ Key schedule (left)
  - generates round keys from cipher key  $K$



# Substitution-permutation network (SPN)



## Substitution-permutation network (SPN)



Round function in data path with two (or three) layers

- ▶ **Non-linear** substitution layer: **S-boxes** applied in parallel
- ▶ permutation layer: moves bits to different S-box positions
- ▶ either key-dependent S-boxes or third layer of *key addition*



## Data encryption standard (DES)

- ▶ Standard by and for US government
- ▶ By National Institute for Standardization and Technology (NIST)
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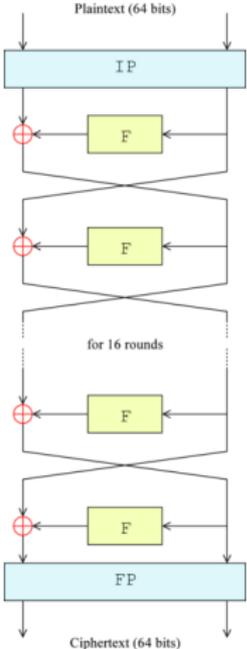
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  - freely usable
- ▶ Massively adopted by banks and industry worldwide
- ▶ Dominated symmetric crypto for more than 20 years

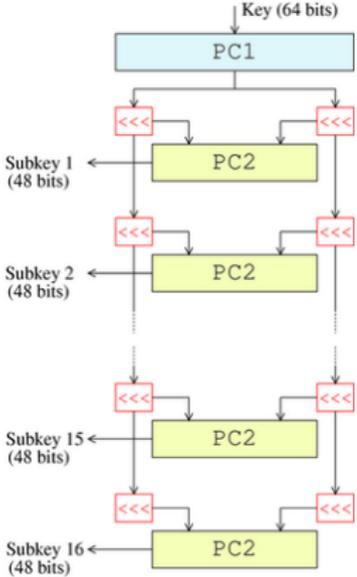


# Data encryption standard: overview

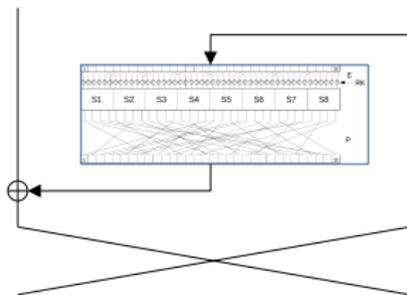
### Feistel data path



### Linear key schedule

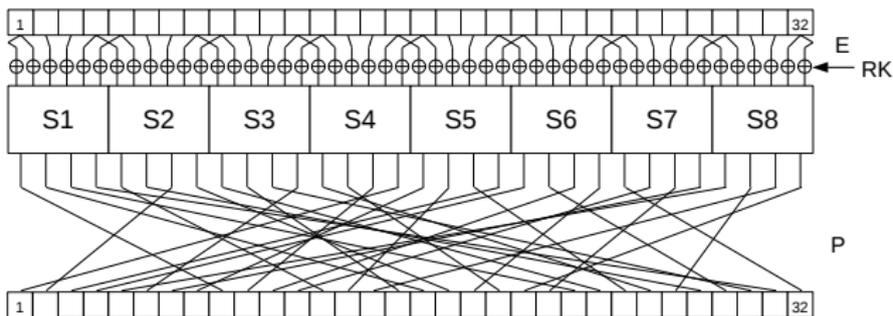


## Data encryption standard: F-function



- ▶ Variant of SPN with 4 layers:

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- ▶ Variant of SPN with 4 layers:
  - expansion E: from 32 to 48 bits
  - bitwise round key addition
  - substitution: 8 different 6-to-4 bit non-linear S-boxes
  - permutation P: moving nearby bits to remote positions
- ▶ clearly hardware-oriented



## Non-ideal DES property: Weak Keys

- ▶ What happens if the cipher key is all-zero?
  - all round keys are all-zero
  - all rounds are the same
  - cipher and its inverse are the same
- ▶ Same is true for an all-one cipher key
- ▶ And two more keys due to symmetry in key schedule
- ▶ Weak key  $K_w$ :

$$\text{DES}[K_w] \circ \text{DES}[K_w] = I$$

- ▶ Also 6 semi-weak key pairs  $(K_1, K_2)$

$$\text{DES}[K_1] \circ \text{DES}[K_2] = I$$

- ▶ Mostly of academic interest



## Non-ideality in DES: Complementation Property

- ▶ What happens if we complement the cipher input?
  - flip all bits in key
  - flip all bits in plaintext
- ▶ In first round
  - input to  $F$  complemented so output of  $E$  complemented
  - round key also complemented so input to S-boxes unaffected
  - output of  $F$  unaffected
- ▶ Output of first round is simply complemented
- ▶ Repeat this until you reach the ciphertext
- ▶ Complementation property:

$$\text{DES}[K](P) = C \Leftrightarrow \text{DES}[\bar{K}](\bar{P}) = \bar{C}$$

- ▶ Reduces complexity of exhaustive key search from  $2^{55}$  to  $2^{54}$



## Non-ideal DES properties: statistical attacks

- ▶ Two specific key-recovery attacks:
  - differential cryptanalysis: exploits difference propagation
  - linear cryptanalysis: exploits large  $P$ -to- $C$  correlations
- ▶ Differential cryptanalysis [Biham and Shamir, 1990]
  - propag. of plaintext difference  $\Delta_p$  to ciphertext difference  $\Delta_c$
  - $DP(\Delta_p, \Delta_c)$ : probability that  $\Delta_p$  results in  $\Delta_c$
  - $\exists \Delta_p, \Delta_c$  with  $DP(\Delta_p, \Delta_c)$  relatively high for all keys
  - requires  $|Q_s| \approx 2^{47}$  (1000 TeraByte) **chosen** plaintexts
- ▶ Linear cryptanalysis [Matsui, 1992]
  - correlation between bits in plaintext  $u_p^T p$  and ciphertext  $u_c^T c$
  - $Corr(u_p, u_c)$ : correlation between  $u_p^T p$  and  $u_c^T c$
  - $\exists u_p, u_c$  with  $Corr(u_p, u_c)$  relatively high for all keys
  - requires about  $|Q_s| \approx 2^{43}$  (64 TeraByte) **known** plaintexts
- ▶ Both *break* DES but still non-trivial to exploit in the field



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- ▶ More than 15 years ago: “software” cracking
  - about 10.000 workstations
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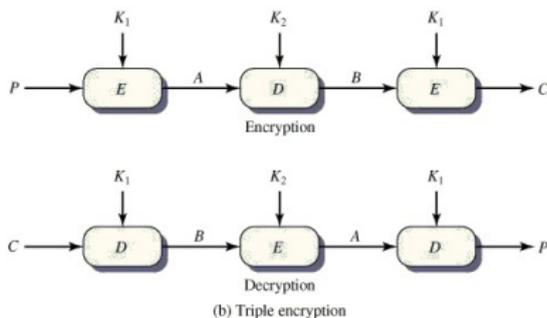


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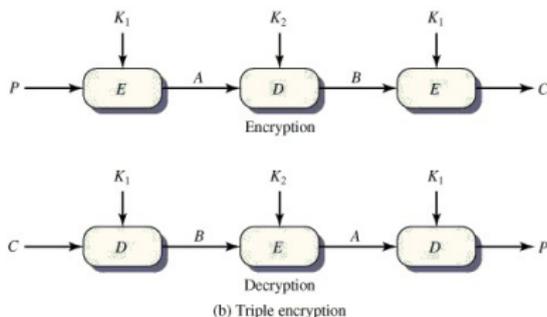
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- ▶ Short DES key is real-world concern!



## The solution: Triple DES (FIPS 46-2 and 46-3)

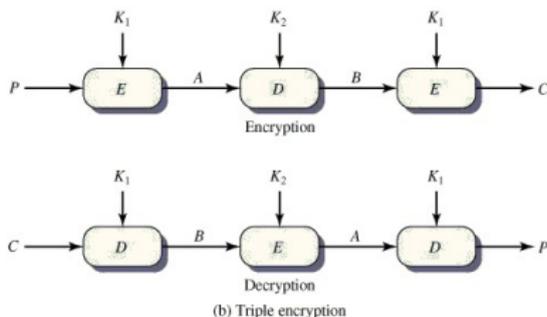


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- Double DES allows meet-in-the-middle attacks

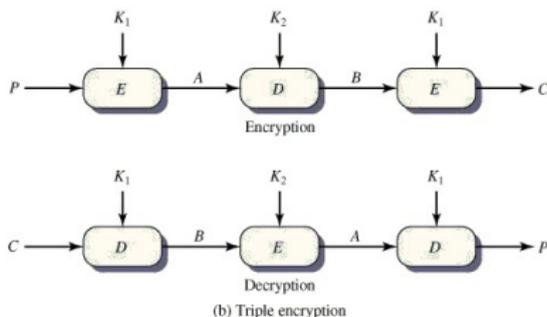
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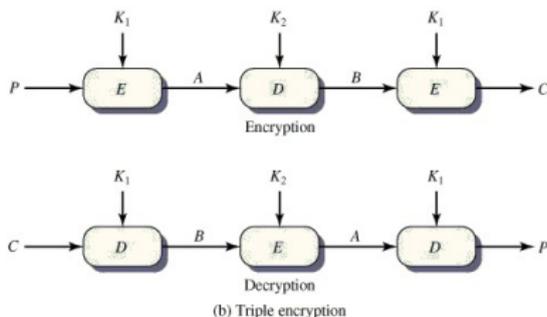
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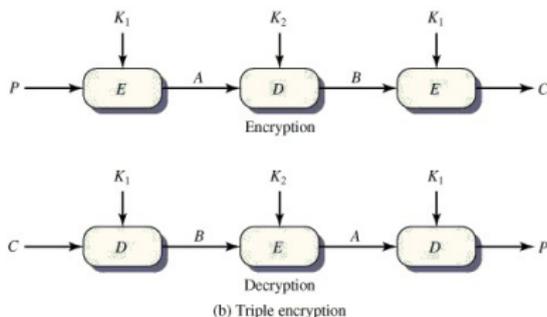
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    - ▶ still massively deployed by banks worldwide
  - 1-key: 56-bit key by taking  $K_3 = K_2 = K_1$ 
    - ▶ falls back to single DES thanks to inverse in middle



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## AES: the result of a competition

- ▶ January 1997: NIST announces the AES initiative
  - replacement of DES
  - open call for block cipher proposals
  - ... and for analysis, comparisons, etc.
- ▶ September 1997: official request for proposals
  - *faster than Triple-DES*
  - 128-bit blocks, 128-, 192- and 256-bit keys
  - specs, reference and optimized code, test vectors
  - design rationale and preliminary analysis
  - patent waiver
- ▶ Vincent Rijmen and I decided to submit a variant of Square
  - Most important change: multiple key and block lengths
  - We call it Rijndael



## The AES competition

- ▶ First round: August 1998 to August 1999
  - 15 candidates at 1st AES conference in Ventura, California
  - analysis presented at 2nd AES conf. in Rome, March 1999
  - NIST narrowed down to 5 finalists using this analysis
- ▶ Second round: August 1999 to summer 2000
  - analysis presented at 3rd AES conf. in New York, April 2000
  - NIST selected winner using this analysis
- ▶ Criteria
  - security margin
  - efficiency in software and hardware
  - key agility
  - simplicity
- ▶ NIST motivated their choice in two reports



## Rijndael design approach: the wide trail strategy

- ▶ Round function with four layers, each with separate goal:
  - nonlinear layer: S-boxes with high non-linearity
  - dispersion layer: like  $P$  in DES  $F$ -function
  - mixing layer (absent in DES): linear local mixing
  - round key addition



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- ▶ Quality of mixing layer quantified its **branch number  $\mathcal{B}$** 
  - allows proving bounds related to resistance against LC/DC
  - in combination with S-box layer and transposition layer
  - link with theory of error-correcting codes
  - optimum mix layer = maximum-distance-separable (MDS) code



## Rijndael

- ▶ Block cipher with block and key lengths  $\in \{128, 160, 192, 224, 256\}$ 
  - set of 25 block ciphers
  - AES limits block length to 128 and key length to multiples of 64



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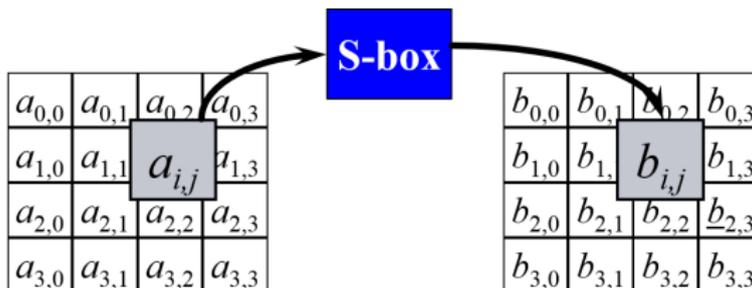


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- ▶ Manipulates bytes with simple operations in  $GF(2^8)$

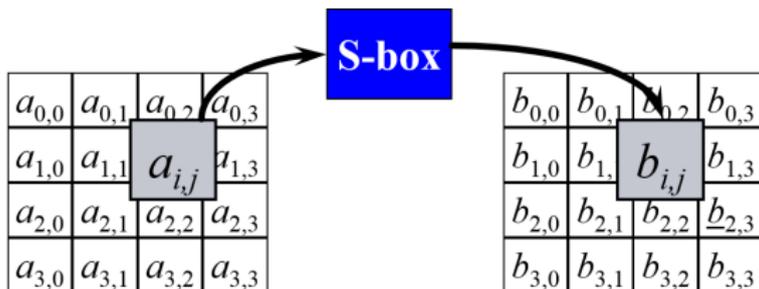


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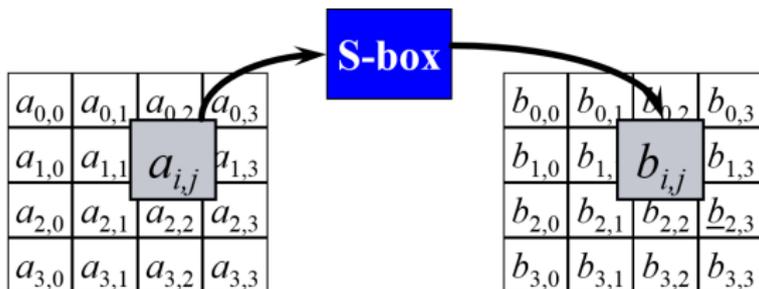


Single S-box with two layers:

- ▶  $y = x^{254}$  in  $GF(2^8)$ 
  - $x^{\#x} = 1$  (Lagrange) so  $y = x^{-1}$  for  $x \neq 0$
  - optimal **non-linearity** [Nyberg, Eurocrypt 1993]



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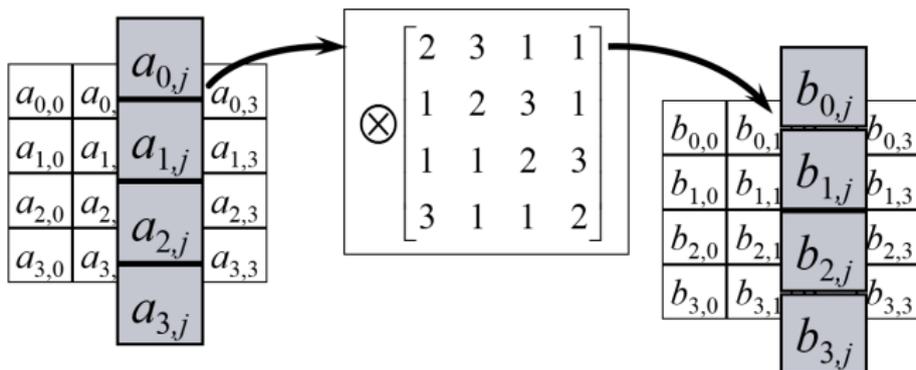


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  - optimal **non-linearity** [Nyberg, Eurocrypt 1993]
- ▶ **Affine mapping**: multiplication by  $8 \times 8$  matrix in  $GF(2)$ 
  - to have algebraic complexity, without it:  $xy = 1$  for  $x \neq 0$

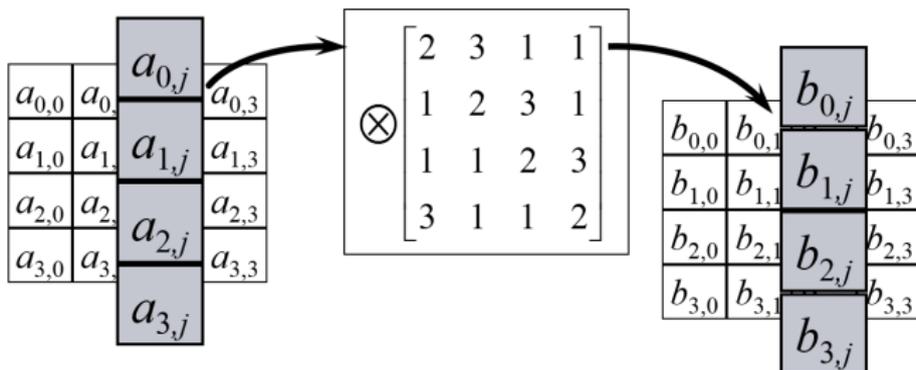


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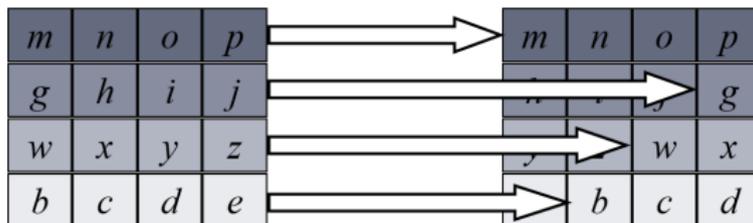
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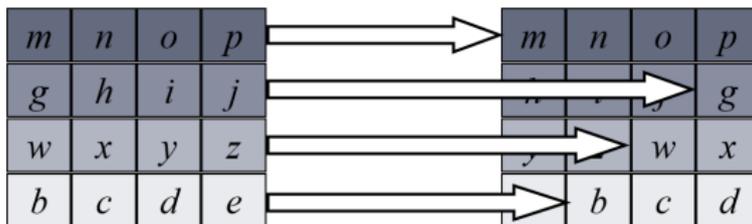


- ▶ Same mapping applied to all 4 columns
- ▶ Multiplication by a  $4 \times 4$  circulant matrix in  $GF(2^8)$ 
  - Elements: 1, 1,  $x$  and  $x + 1$
  - *circulant MDS ( $\mathcal{B} = 5$ ) matrix with the simplest elements*
  - Inverse has more complex elements

## The dispersion layer: ShiftRows



## The dispersion layer: ShiftRows



- ▶ Each row is shifted by a different amount
- ▶ Different shift offsets for higher block lengths
- ▶ Together with MixColumns and SubBytes:
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  - full diffusion in two rounds
  - $B^2 = 25$  active *S*-boxes in 4 rounds

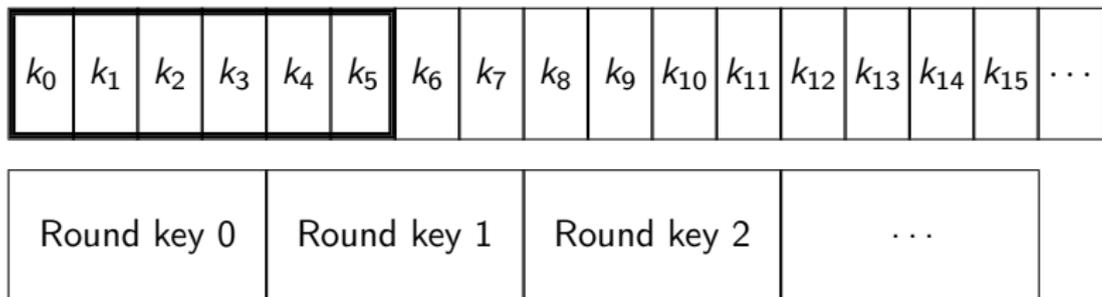


# Round key addition: AddRoundKey

$$\begin{array}{|c|c|c|c|} \hline a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ \hline a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ \hline a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ \hline a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline k_{0,0} & k_{0,1} & k_{0,2} & k_{0,3} \\ \hline k_{1,0} & k_{1,1} & k_{1,2} & k_{1,3} \\ \hline k_{2,0} & k_{2,1} & k_{2,2} & k_{2,3} \\ \hline k_{3,0} & k_{3,1} & k_{3,2} & k_{3,3} \\ \hline \end{array} = \begin{array}{|c|c|c|c|} \hline b_{0,0} & b_{0,1} & b_{0,2} & b_{0,3} \\ \hline b_{1,0} & b_{1,1} & b_{1,2} & b_{1,3} \\ \hline b_{2,0} & b_{2,1} & b_{2,2} & b_{2,3} \\ \hline b_{3,0} & b_{3,1} & b_{3,2} & b_{3,3} \\ \hline \end{array}$$



## Key schedule: 192-bit key, 128-bit block example

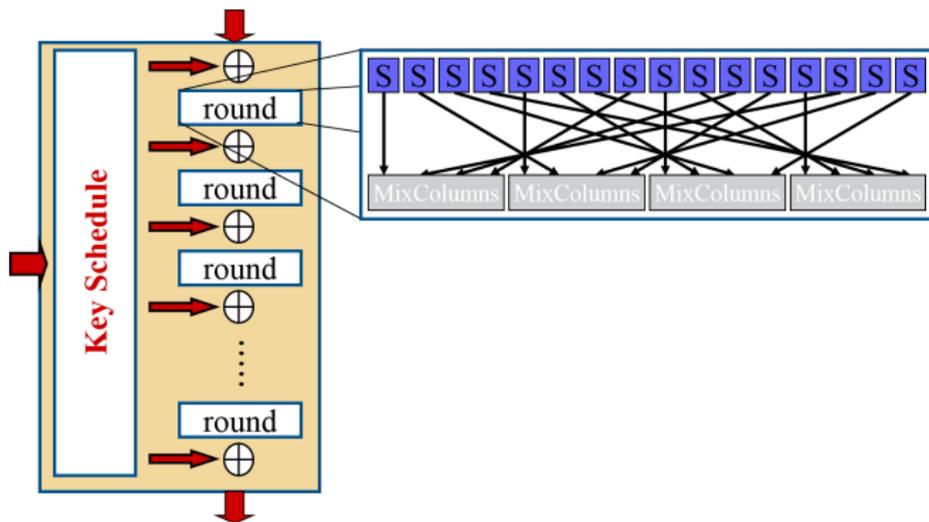


$$k_{6n} = k_{6n-6} \oplus f(k_{6n-1})$$
$$k_i = k_{i-6} \oplus k_{i-1}, \quad i \neq 6n$$

$f$ : AES S-box in parallel to 4 bytes followed by cyclic shift over 1 byte



## Rijndael: summary



- ▶ # rounds:  $6 + \max(\ell_k, \ell_b)$  with  $\ell_k$  key and  $\ell_b$  block length in 32-bit words
- ▶ last round has no MixColumns to make inverse similar to cipher

## Rijndael symmetry

- ▶ Highly symmetric round function (as opposed DES)
  - SubBytes: 1 S-box instead of different ones
  - MixColumns: 1 MDS matrix with circulant symmetry
  - ShiftRows: bytes relative movement independent of position
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- ▶ Very high symmetry in nonlinear part of S-box:  $y = x^{-1}$ 
  - representation of elements of  $GF(2^8)$ : choice of **basis**
  - elements as degree  $< 2$  polynomials with coeff. in  $GF(2^4)$
  - can be done recursively
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  - can be done recursively
  - called **tower fields**
- ▶ Asymmetry:
  - inverse is different and slightly more expensive
  - key schedule has some symmetry, but much less



## Rijndael implementation aspects

- ▶ Implementations can exploit symmetry



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- ▶ Software with table-lookups:
  - 4 Kbytes of table
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- ▶ Software in bitslice:
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  - only bitwise Boolean instructions and shifts
- ▶ Hardware:
  - very suitable thanks to arithmetic in  $GF(2^n)$  instead of  $(\mathbb{Z}_{2^n}, +)$
  - fully parallel: combinatorial logic with full round
  - serial: logic for 1 S-box and 1 MixColumns matrix column
  - S-box area/circuit depth trade-off by using tower fields



## Rijndael security status

- ▶ Cryptanalysis (in public domain)
  - all attacks, also on reduced-round, have huge data complexity
  - there is an (academic) attack against full-round AES:
    - ▶ biclique attacks [Bogdanov, Khovratovich, Rechberger, 2011]
    - ▶  $|Q_c| \approx 2^{126}$ : factor 2 gain compared to exhaustive key search
    - ▶ gain evaporates when looking at complete picture
  - solid security status thanks to public scrutiny
- ▶ Implementation attacks: exploiting implementation weaknesses
  - timing attacks: cache misses in table-lookups
  - power analysis: exploiting dependence of current on data
  - electromagnetic analysis: same for EM emanations
  - fault attacks: exploiting forced faults
- ▶ Implementation attacks are the ones that matter in practice!



# Currently we are here...

Introduction

Block cipher model and security definition

Data Encryption Standard (DES)

Advanced Encryption Standard (AES)

Encryption modes of block ciphers

Authentication modes of block ciphers



## Block cipher modes for encryption

- ▶ DES can encipher 8-byte messages, AES of 16-byte messages
  - what about longer and shorter messages?
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- ▶ Stream encryption modes
  - build a stream cipher with a block cipher as building block



## Block encryption modes

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  - each cryptogram bit depends on each message bit and vice versa
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- ▶ Due to padding, cryptogram is longer than message



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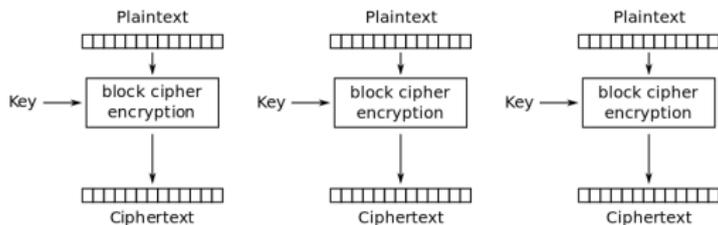


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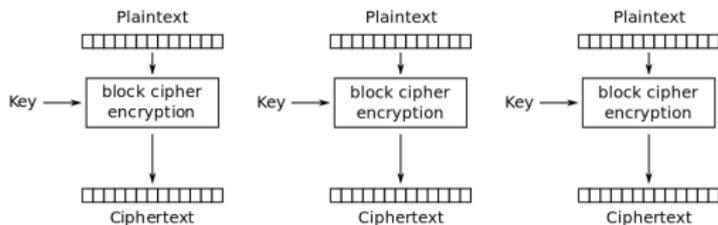


Electronic Codebook (ECB) mode encryption

- ▶ Advantages
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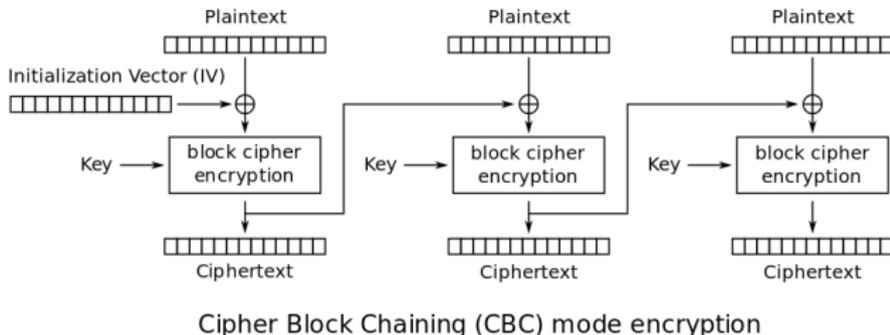


Electronic Codebook (ECB) mode encryption

- ▶ Advantages
  - simple
  - parallelizable
- ▶ Limitation: equal plaintext blocks  $\rightarrow$  equal ciphertext blocks:
  - likely to happen in low-entropy messages
  - problem in padded last block



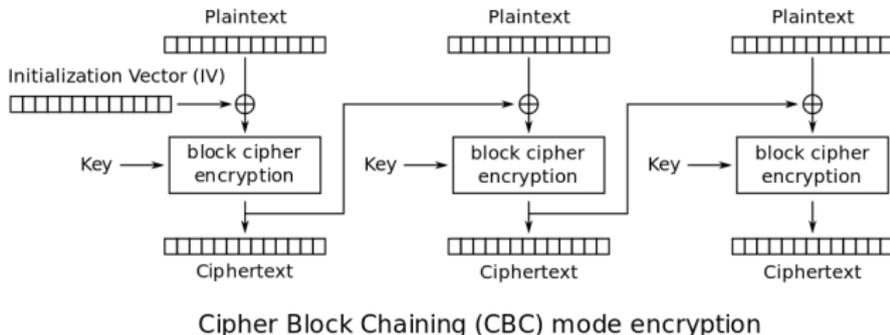
## Cipher Block Chaining mode (CBC)



- ▶ *ECB with plaintext block randomized by previous ciphertext block*



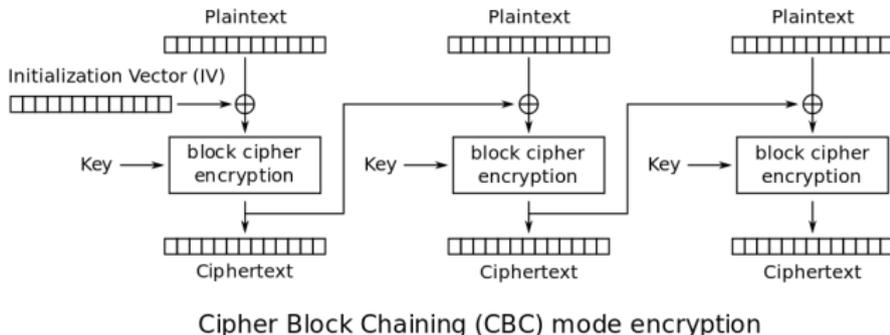
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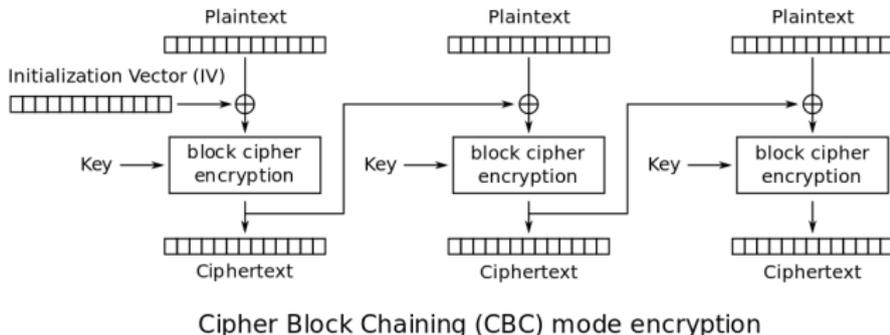
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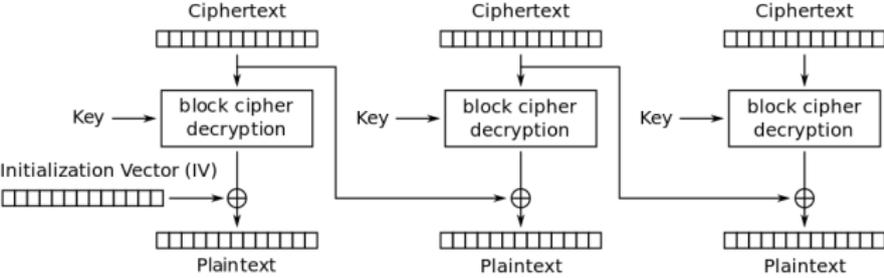
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  - requires randomly generating and transferring *IV*



# Cipher Block Chaining mode (cont'd)

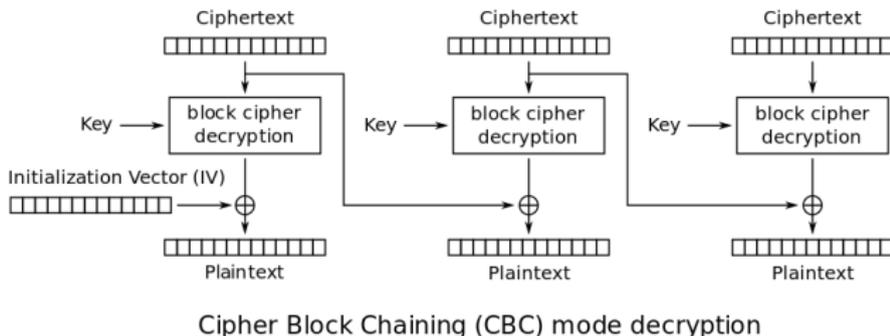


Cipher Block Chaining (CBC) mode decryption

- ▶ Replacing  $IV$  randomness by  $N$  nonce requirement:  $IV = B[K](N)$



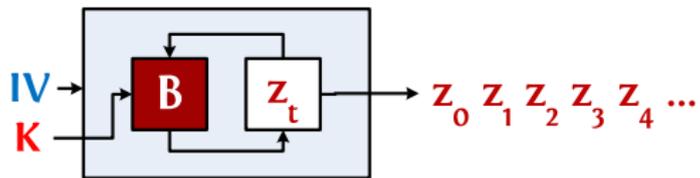
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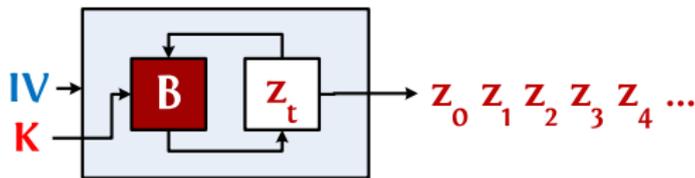
- ▶ Replacing  $IV$  randomness by  $N$  nonce requirement:  $IV = B[K](N)$
- ▶ Properties of CBC
  - encryption strictly serial, decryption can be parallel
  - $IV$  must be managed and transferred
  - security less than what one would think



## Stream encryption: Output FeedBack mode (OFB)



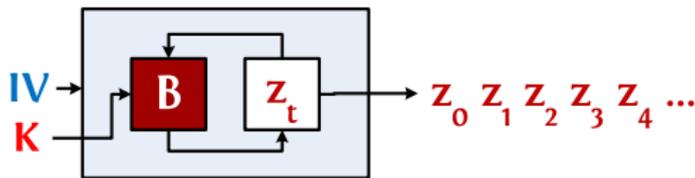
## Stream encryption: Output FeedBack mode (OFB)



- ▶ Stream cipher with:
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  - initialization:  $z_{-1} = IV$
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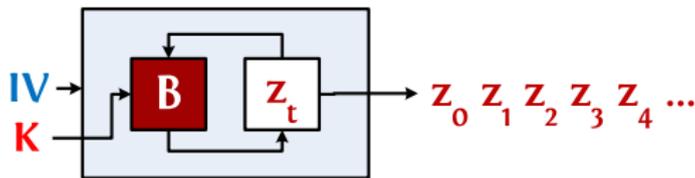
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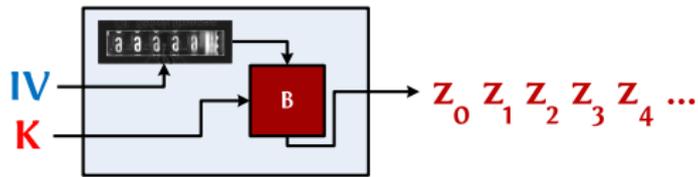
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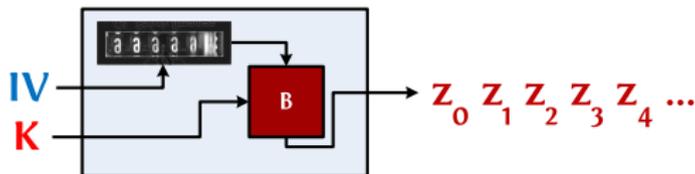
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  - output: just take  $z_t$
- ▶ Properties
  - strictly serial
  - cycle lengths not known in advance
  - no need for  $B^{-1}$  (valid for all stream encryption)



## Stream encryption: Counter mode



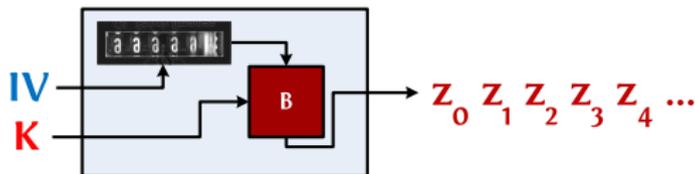
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- ▶ Stream cipher with:
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  - output:  $z_t = B[K](c^t)$



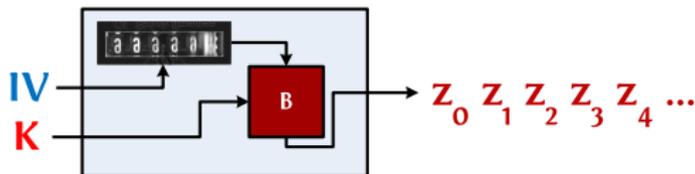
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  - output:  $z_t = B[K](c^t)$
- ▶ Properties
  - fully parallelizable
  - cycle length  $2^b$  with  $b$  the block length



## Encryption modes: overview

	ECB	CBC	OFB	Counter
parallel encryption	y	n	n	y
parallel decryption	y	y	n	y
random access	y	y	n	y
requires $B^{-1}$	y	y	n	n
requires padding	y	y	n	n
full collapse if nonce violation	n	n	y	y
error propagation $C \rightarrow P$	y	y	n	n

Legend:

- ▶ random access: fast decryption of bits anywhere in the message
- ▶ error propagation: single-bit error in  $C$  expands to  $b$  bits in  $P$



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Block cipher model and security definition

Data Encryption Standard (DES)

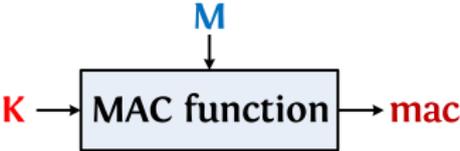
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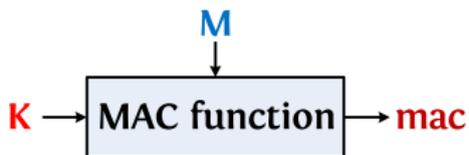
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# Message authentication code (MAC) functions



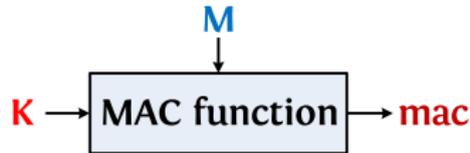
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  - input: key  $K$  and arbitrary-length message  $M$
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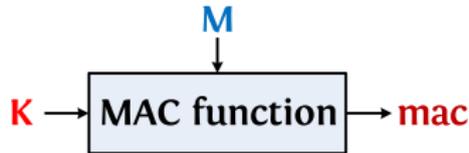
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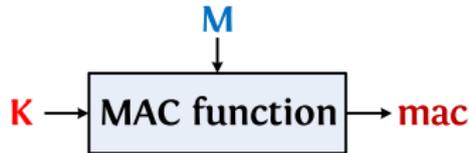
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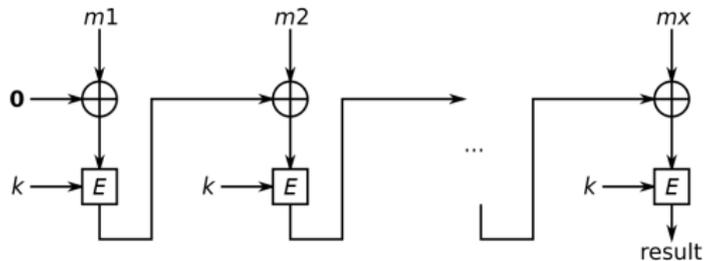
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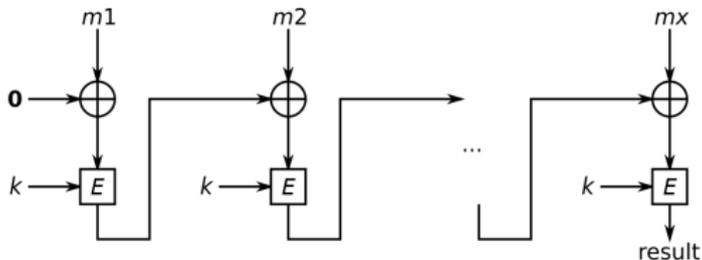
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- ▶ If ideal,  $\Pr(\text{success})$  of forging a pair  $M, T = \text{MAC}(K, M)$  is  $2^{-\ell}$



# Cipher Block Chaining MAC mode (CBC-MAC)



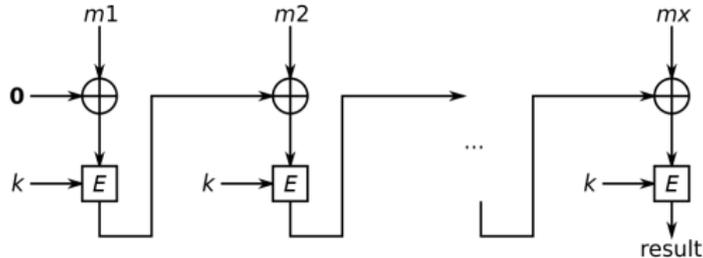
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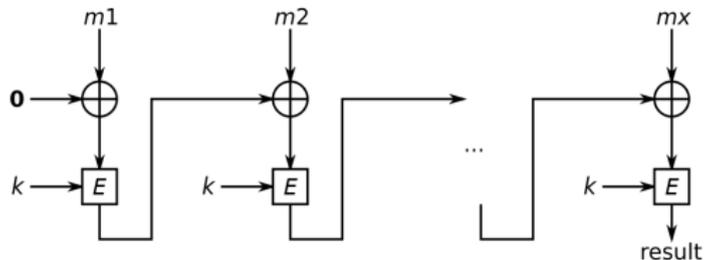
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- ▶ Idea:
  - Apply CBC encryption to (padded) message
  - take  $T$  equal to last ciphertext block
  - throw away other blocks (essential for security)



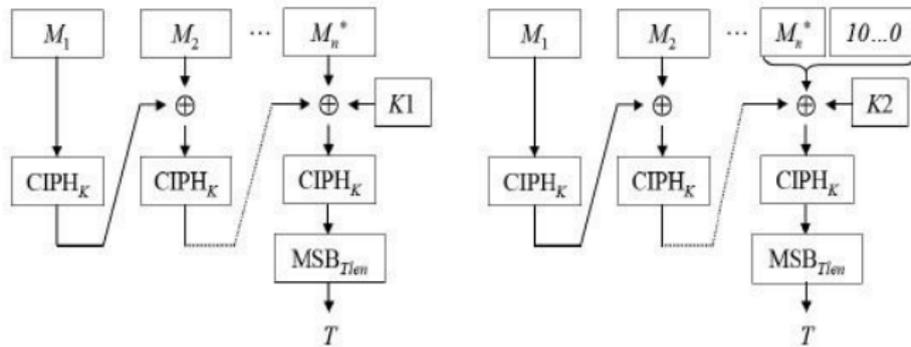
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- ▶ Idea:
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  - throw away other blocks (**essential for security**)
- ▶ Broken for arbitrary-length messages
  - **length-extension weakness**



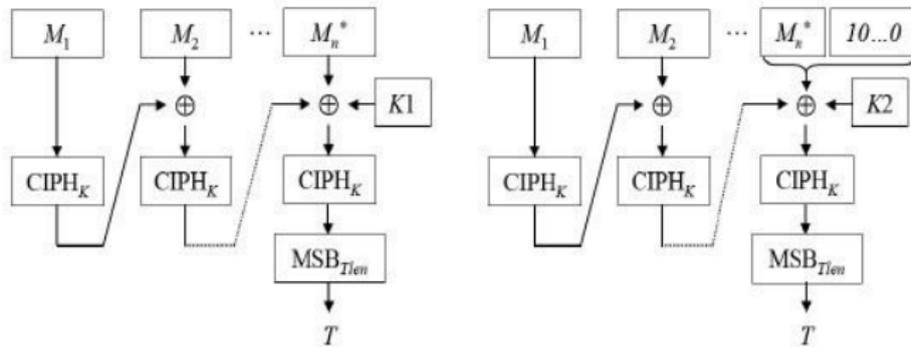
## A fix of CBC-MAC: C-MAC



- NIST standard: Special Publication 800-38B



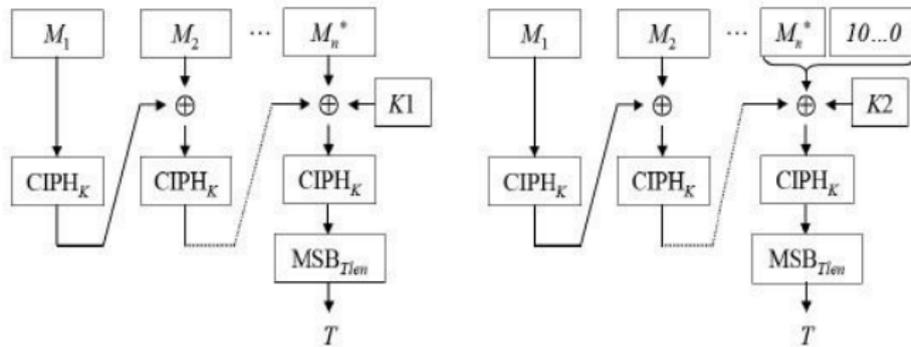
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- ▶ avoid length-extension by *doing something different at the end*



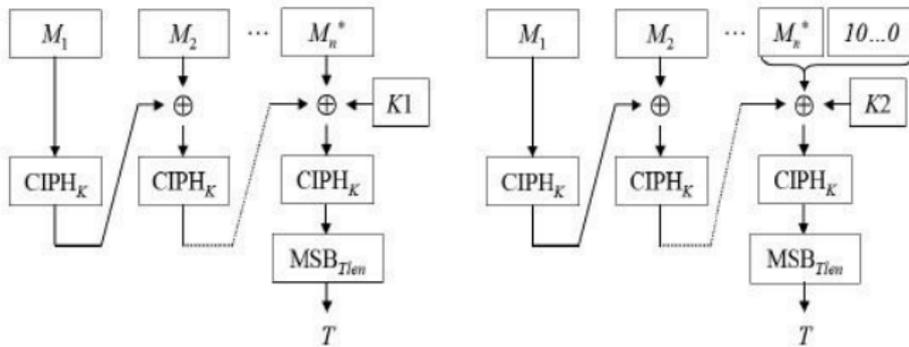
## A fix of CBC-MAC: C-MAC



- ▶ NIST standard: Special Publication 800-38B
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- ▶ addition of a constant before last application of  $B[K]$



## A fix of CBC-MAC: C-MAC



- ▶ NIST standard: Special Publication 800-38B
- ▶ avoid length-extension by *doing something different at the end*
- ▶ addition of a constant before last application of  $B[K]$
- ▶ Secure for arbitrary-length messages



# Currently we are here...

Introduction

Block cipher model and security definition

Data Encryption Standard (DES)

Advanced Encryption Standard (AES)

Encryption modes of block ciphers

Authentication modes of block ciphers



## Summary

- ▶ Block ciphers are keyed  $b$ -bit permutations
  - a different permutation  $B[K]$  per key  $K$  (and tweak  $w$ )
  - with an efficient inverse  $B[K]^{-1}$
  - exhaustive keysearch should be best attack (complexity  $2^{|K|-1}$ )
- ▶ DES and AES are the most widespread block ciphers
  - constructed by iterating a simple round function
  - round has steps for non-linearity, mixing and transposition
- ▶ Block ciphers are versatile:
  - block encryption modes: e.g., ECB and CBC
  - stream encryption modes: e.g., OFB, counter and CFB
  - MAC computation modes: e.g., CBC-MAC and C-MAC
- ▶ Inverse permutation only used in block encryption modes

