Message Authentication Codes (MACs)

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About Me

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Outline

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- Introduction

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- HMAC
- Universal-hash based MACs
  - Poly1305
  - security issues
  - software implementation issues

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  - software implementation issues
- Diffie–Hellman key exchange

What are MACs?

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- On Wikipedia:

> *"a message authentication code (often MAC) is a short piece of information used to authenticate a message and to provide integrity and authenticity assurances on the message. Integrity assurances detect accidental and intentional message changes, while authenticity assurances affirm the message's origin"*

Digital Signatures

Digital Signatures

- Construction:

$$\text{message } (m) \longrightarrow \boxed{\text{hash}} \longrightarrow h \quad \boxed{\text{TP}} \quad \text{signature } (s)$$

$$h \xrightarrow{sk} s, \qquad s \xrightarrow{pk} h$$

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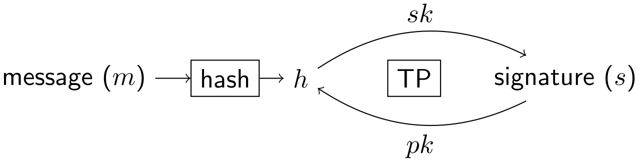

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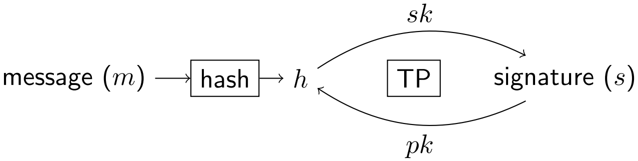

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✍ secret-key crypto is "fast"

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$$\begin{tikzcd}
 & m_1 \arrow[d] & m_2 \arrow[d] & & m_\ell \arrow[d] & \\
IV \arrow[r] & \boxed{f} \arrow[r] & \boxed{f} \arrow[r] & \cdots \arrow[r] & \boxed{f} \arrow[r] & h
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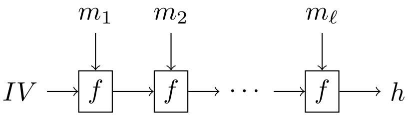

- Length extension attack: $h' = f(h, m_{\ell+1})$

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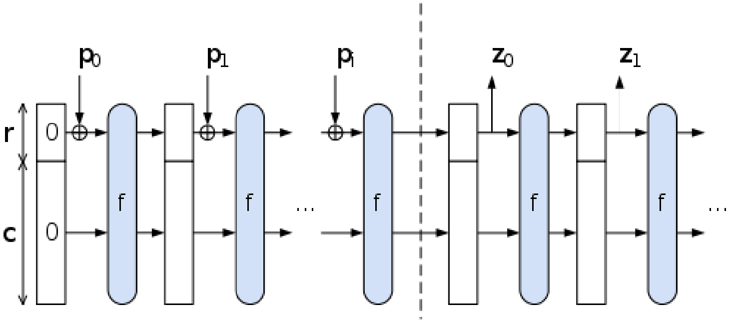
✍ Reality: the most commonly used scheme might not be the best

SHA3

The "Sponge" construction:



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  - universal hash: low differential probability
  - one-time pad hides all information about the key

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- Speed: 1.22 Sandy Bridge cycles/byte

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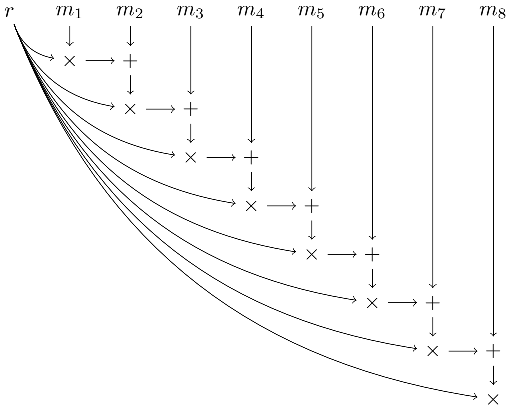

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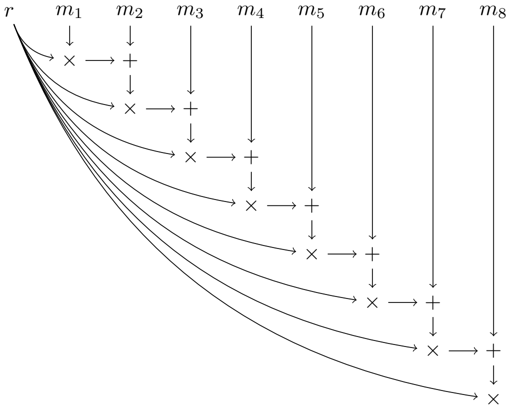

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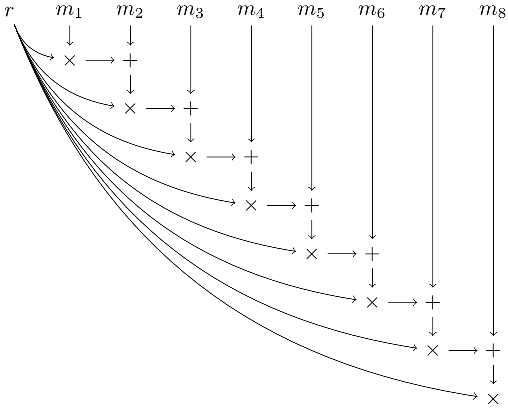

- n multiplications (and $n-1$ additions)
- The issue of being *"on-line"*

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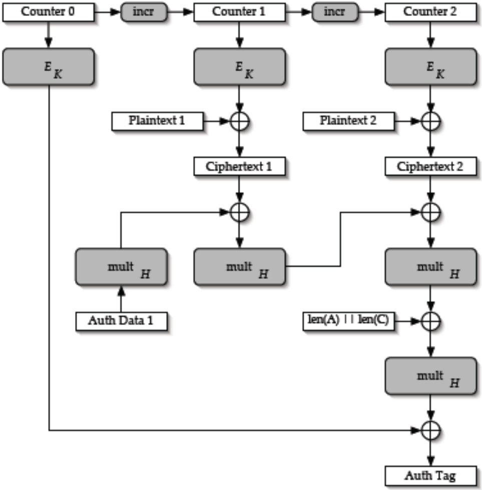
✍ Binary fields: better in hardware

GCM



http://en.wikipedia.org/wiki/Galois/Counter_Mode

GMAC: speeds

reference	platform	`PCLMUQDQ`	cycles per byte
Käsper–Schwabe 2009	Core 2	no	14.40
	Sandy Bridge	no	13.10
Krovetz–Rogaway 2011	Westmere	yes	2.00
Gueron 2013	Sandy Bridge	yes	1.79
	Haswell	yes	0.40

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  - base field $\mathbb{F}_{2^{256}} = \mathbb{F}_{2^8}[x]/(\phi)$. Tower field construction for $\mathbb{F}_{2^8}$.
- Compared to GMAC
  - higher security level
  - 0.5/1 multiplications per block

Auth256*

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Wegman–Carter construction: security

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- The one-time pad hides all information about the key r.
- The best strategy for the attacker is to guess.

Auth256: Security Proof

Hash values:

$$\begin{aligned} h =& (m_1 + r_1)(m_2 + r_2) + (m_3 + r_3)(m_4 + r_4) + \cdots + (m_{2\ell-1} + r_{2\ell-1})(m_{2\ell} + r_{2\ell}), \\ h' =& (m'_1 + r_1)(m'_2 + r_2) + (m'_3 + r_3)(m'_4 + r_4) + \cdots + (m'_{2\ell-1} + r_{2\ell-1})(m'_{2\ell} + r_{2\ell}). \end{aligned}$$

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Then $h = h' + \Delta$ if and only if

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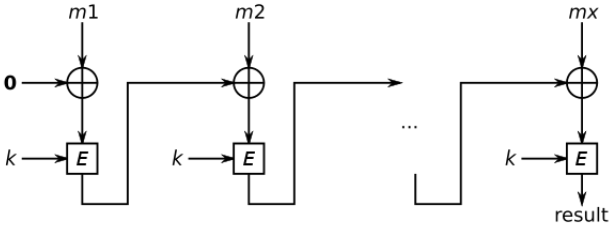
$m \neq m'$ implies that there are at most $|K|^{2\ell-1}$ solutions for r.

CBC-MAC



http://en.wikipedia.org/wiki/CBC-MAC