Proeftentamen 2009-10-12

1. A *boolean ring* is a ring satisfying $x^2 = x$ for all x. Show that a boolean ring is commutative.

2. Compute in $\mathbb{N}$ the product of all even prime numbers larger than 10.

3. Define on $\mathbb{R}$ the operation $*$ by

$$x * y := (x - 1)(y - 1) + 1.$$

Is this operation commutative? Associative? Is there a neutral element? Is R together with $*$ and possibly further operations a semigroup? a monoid? a group? Why (not)?

4. How many monic polynomials of degree 3 are there in $\mathbb{F}_3[X]$? How many of these are irreducible?

5. Determine the size of the automorphism group of the graph below, the 1-skeleton of a cube (8 vertices, 12 edges).