Diffie-Hellman key exchange

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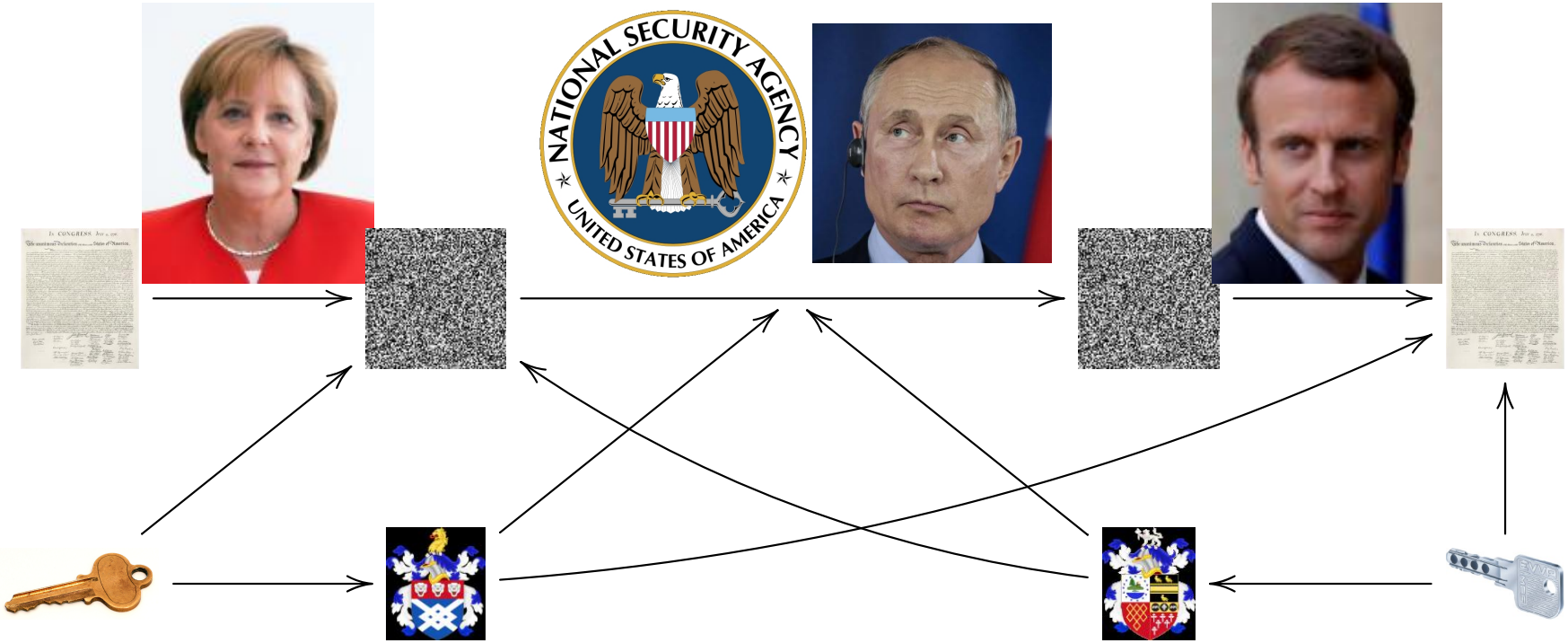
2WF80: Introduction to Cryptology

Public-key authenticated encryption ("DH" data flow)



- Prerequisite: Alice has a private key  and public key  .
- Prerequisite: Bob has a private key  and public key  .
-  and  determine the same key as  and 
- Alice and Bob use this shared key to authenticate and encrypt with symmetric cryptography.

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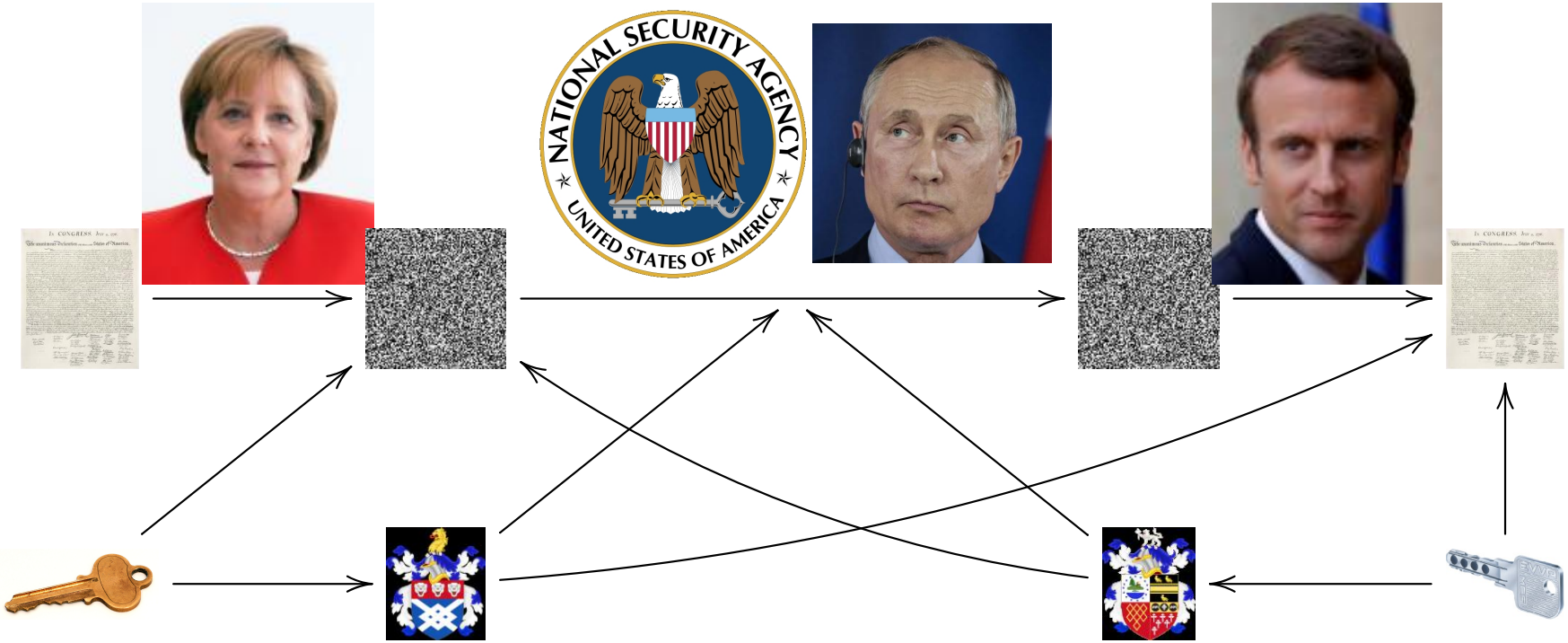

- Everybody knows a group G generator g.
- Prerequisite: Alice has a private key $a \in \mathbb{N}$ and public key $h_A = g^a$.
- Prerequisite: Bob has a private key $b \in \mathbb{N}$ and public key $h_B = g^b$.
- $h_B^a = (g^b)^a$
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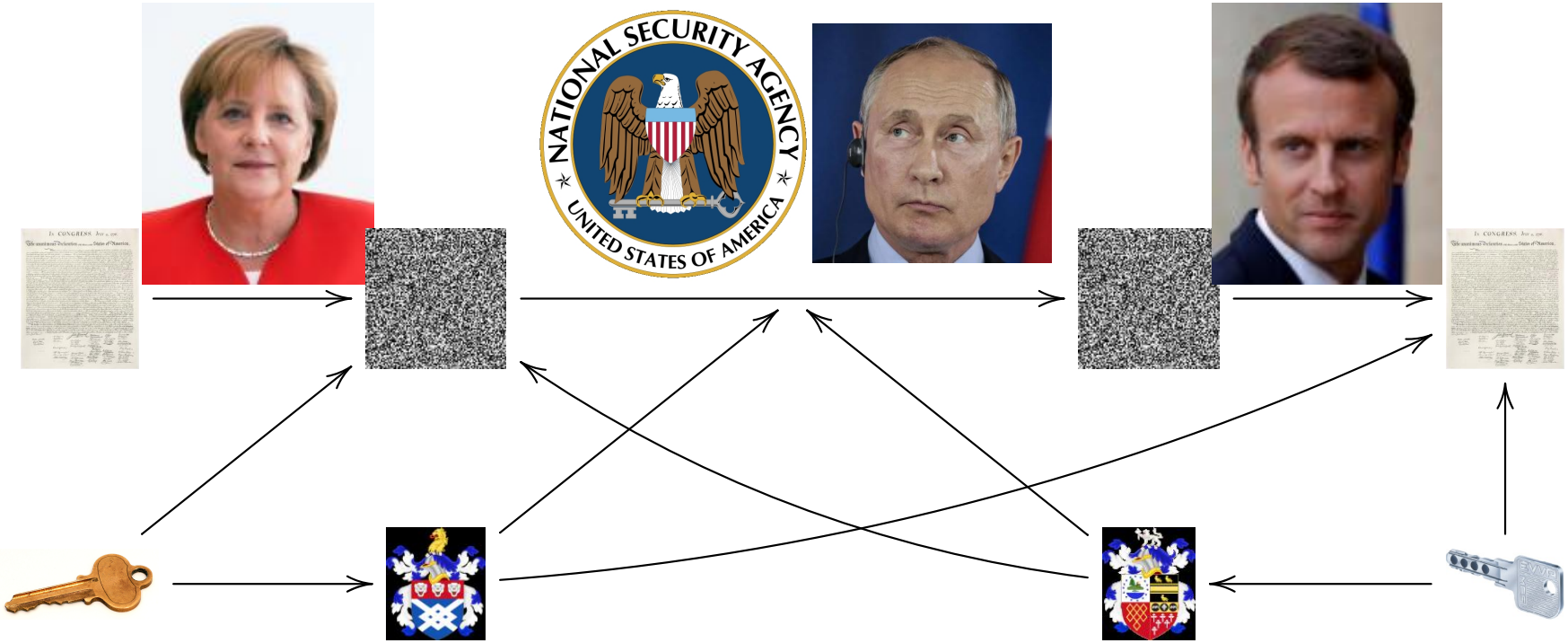

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- $h_B^a = (g^b)^a = g^{ab} = (g^a)^b = h_A^b$. Use hash of g^{ab} has key.
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Diffie–Hellman key exchange

- 1976 Diffie and Hellman introduce public-key cryptography.
- To use it, standardize group G and $g \in G$.
  Everybody knows G and g as well as how to compute in G.
- Warning #1: Many G are unsafe!

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  - $G = (\mathbb{F}_p, +)$, i.e., A sends $h_A \equiv ag \bmod p$.
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- Used in practice $G \subset (\mathbb{F}_p^*, \cdot)$ with g an element of large prime order.
- More commonly used in practice G is group of points on an elliptic curve over $\mathbb{F}_p$. Stay on for 2MMC10 for details.

Hardness assumptions

- Computational Diffie-Hellman Problem (CDHP):
  Given g, g^a, g^b compute g^{ab}.
- Decisional Diffie-Hellman Problem (DDHP):
  Given g, g^a, g^b, and g^c decide whether $g^c = g^{ab}$.
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Practical problems

- Eve can set up a *man-in-the-middle* attack:

$$A \xleftrightarrow{g^{ae}} E \xleftrightarrow{g^{bf}} B$$

  E decrypts everything from A and reencrypts it to B and vice versa.
- This attack cannot be detected unless A and B have some long-term keys that are known to each other or compare their keys out of band.

Semi-static DH

- A cryptosystem combining public-key and symmetric-key crypto is called a *hybrid system*
- Alice publishes long-term public key $h_A = g^a$,
  keeps long-term private key a.
- Any user can encrypt to Alice using this key:
  - Pick random k and compute $r = g^k$.
  - Encrypt message using symmetric keys derived from $H(h_A^k)$.
  - Send ciphertext c along with r.
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- Alice's key here is static, Bob's key is ephemeral.
- Note: ephemeral does not mean one-time;
  it means that is not long term.