Historical ciphers I

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2WF80: Introduction to Cryptology

Caesar cipher

Most famous historical cipher, here with our current alphabet.

```
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
| | |                                           |
D E F G H I J K L M N O P Q R S T U V W X Y Z A B C
```

Encryption just maps letters from the top row to the matching ones in the bottom row.

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HELLO BOB
KHOOR ERE
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Encryption just maps letters from the top row to the matching ones in the bottom row.

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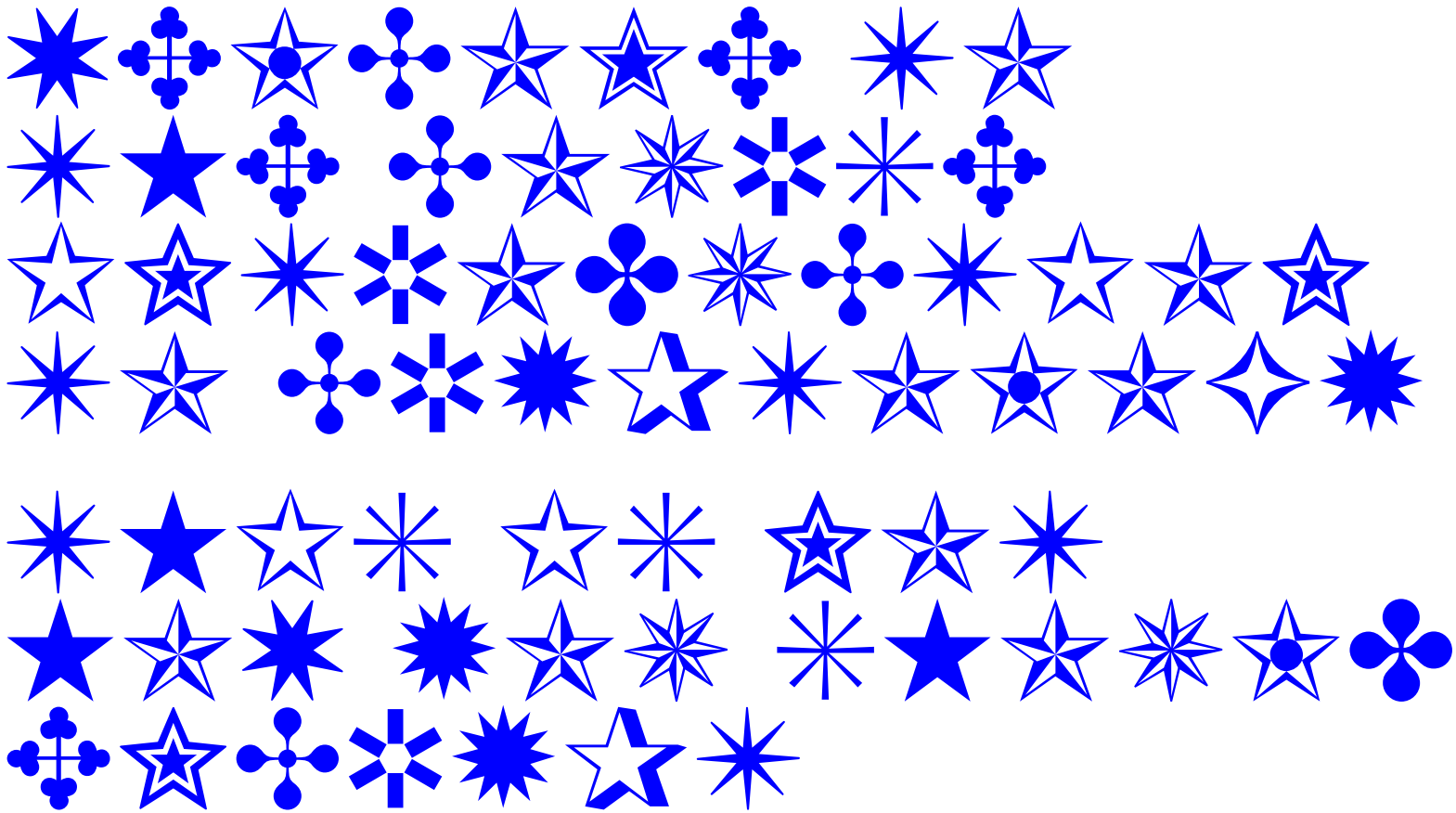
Decryption maps from bottom row to top row.
Figure out what this decrypts to:

```
HQFUBSWLRQ ZRUNV
```

Substitution cipher

Each letter is replaced by a symbol.



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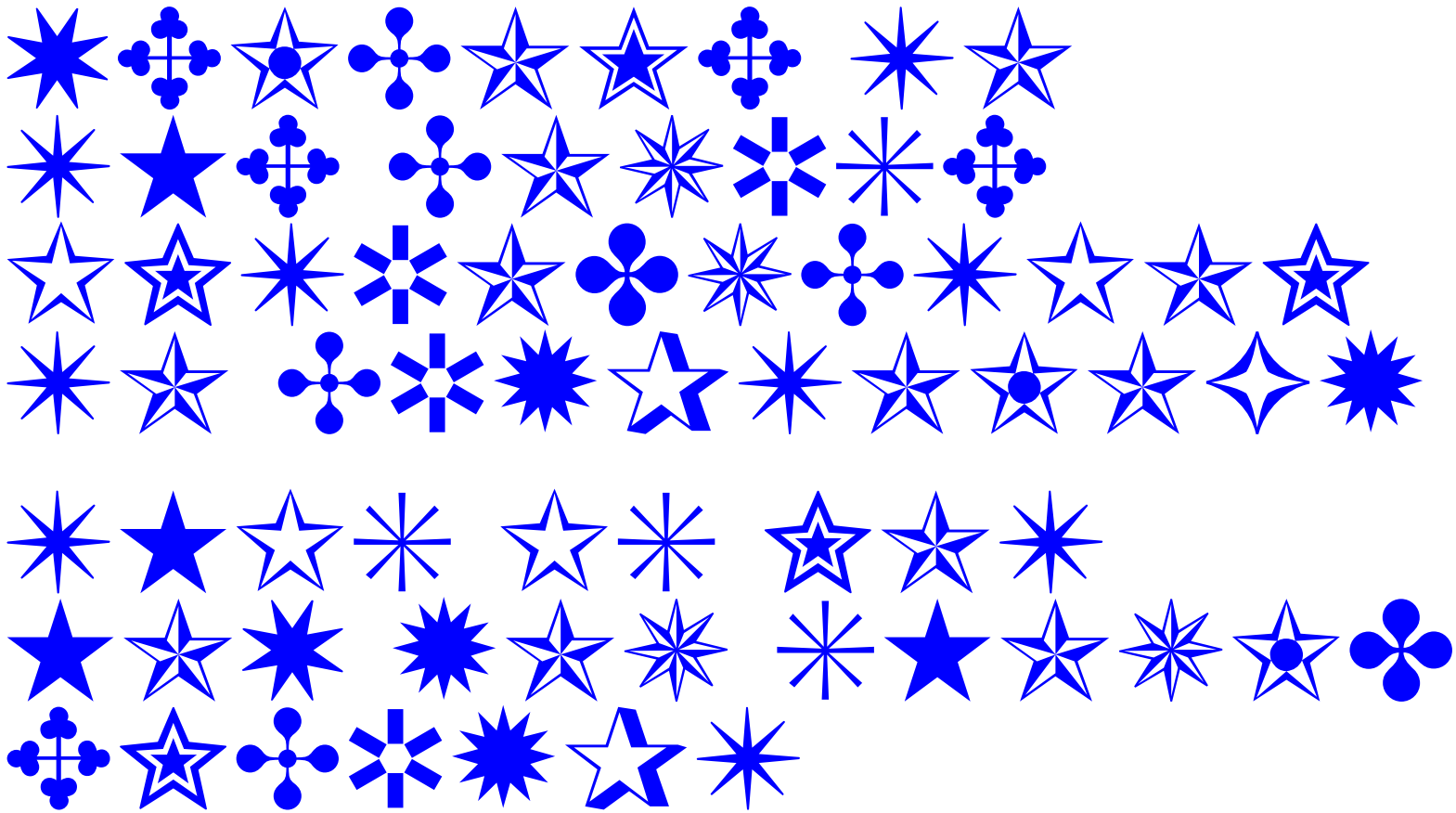

Hint: look for short words and repeated combinations of letters.

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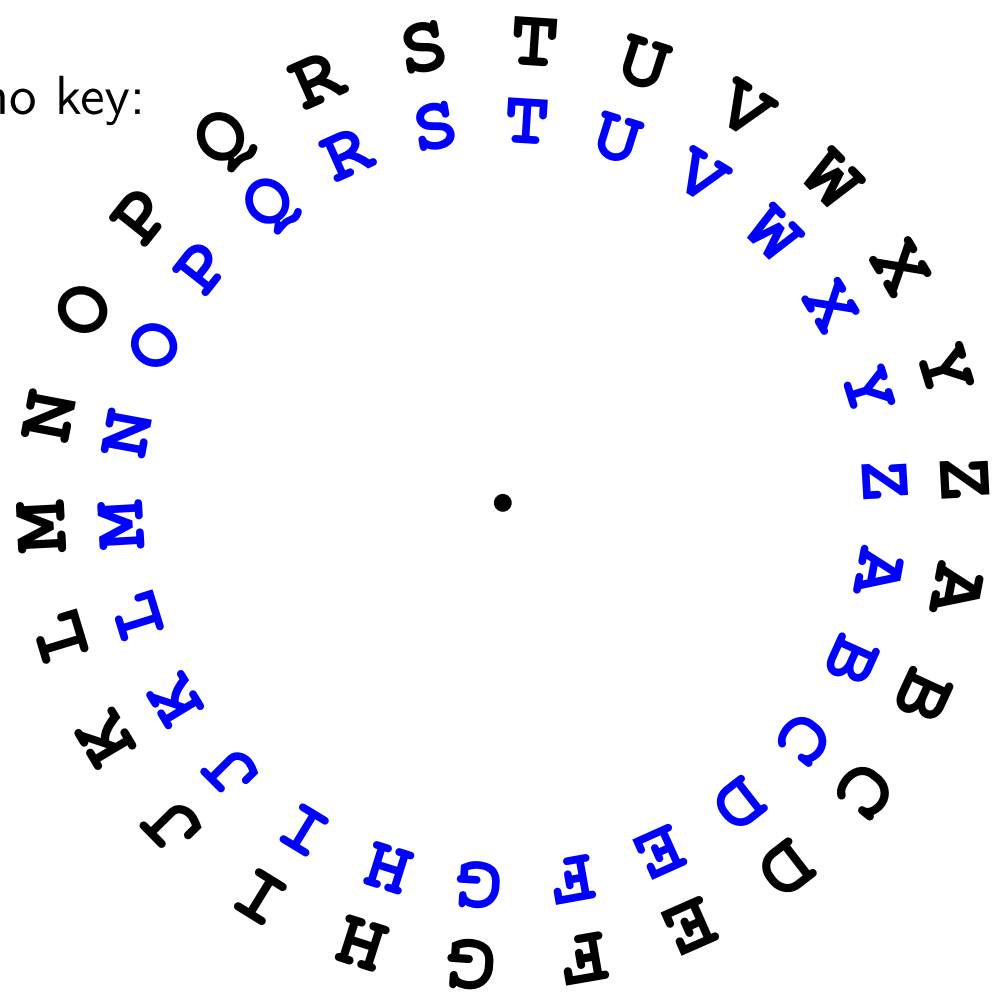

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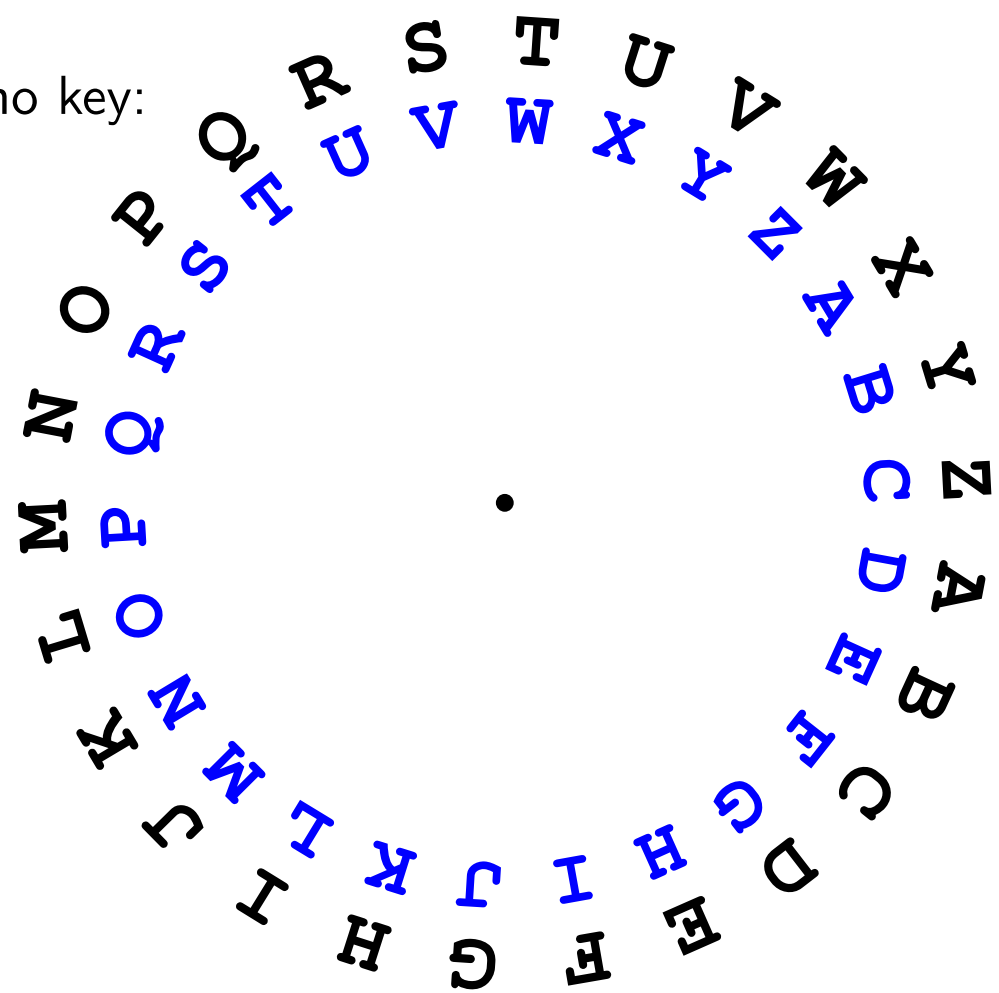

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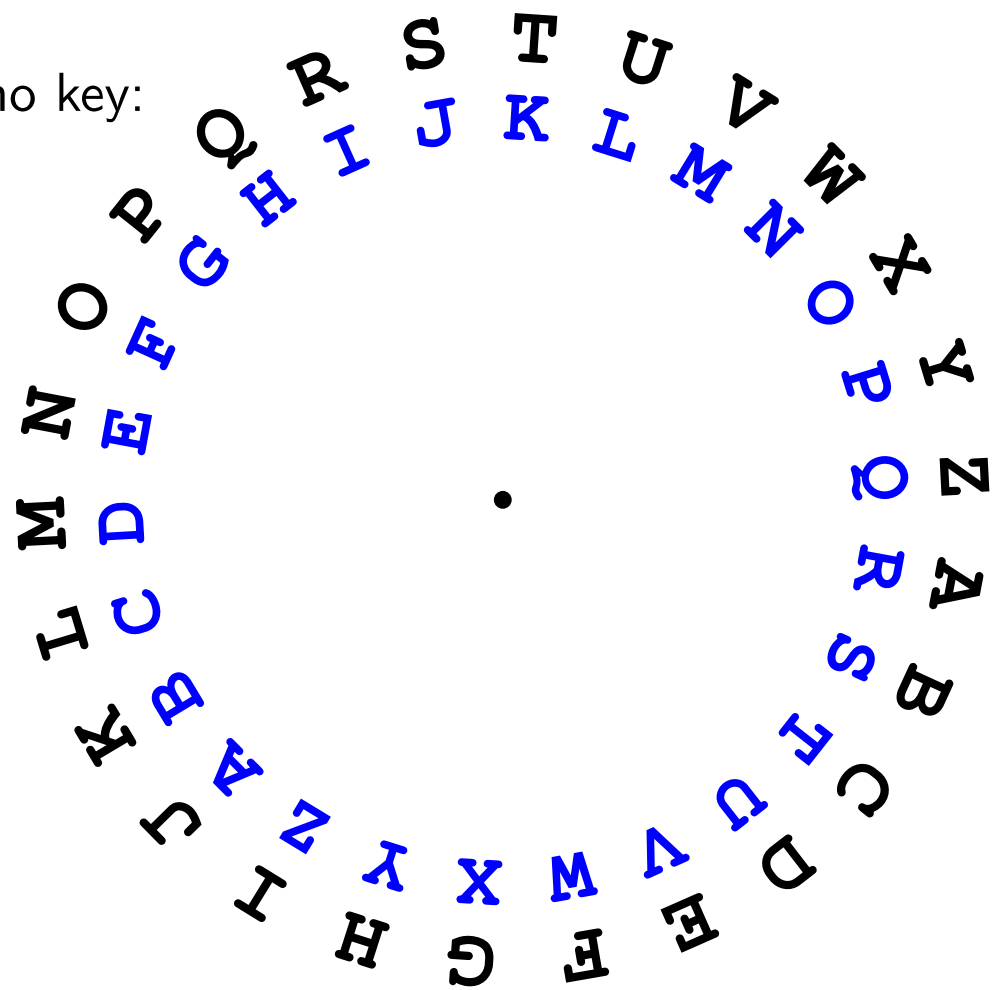

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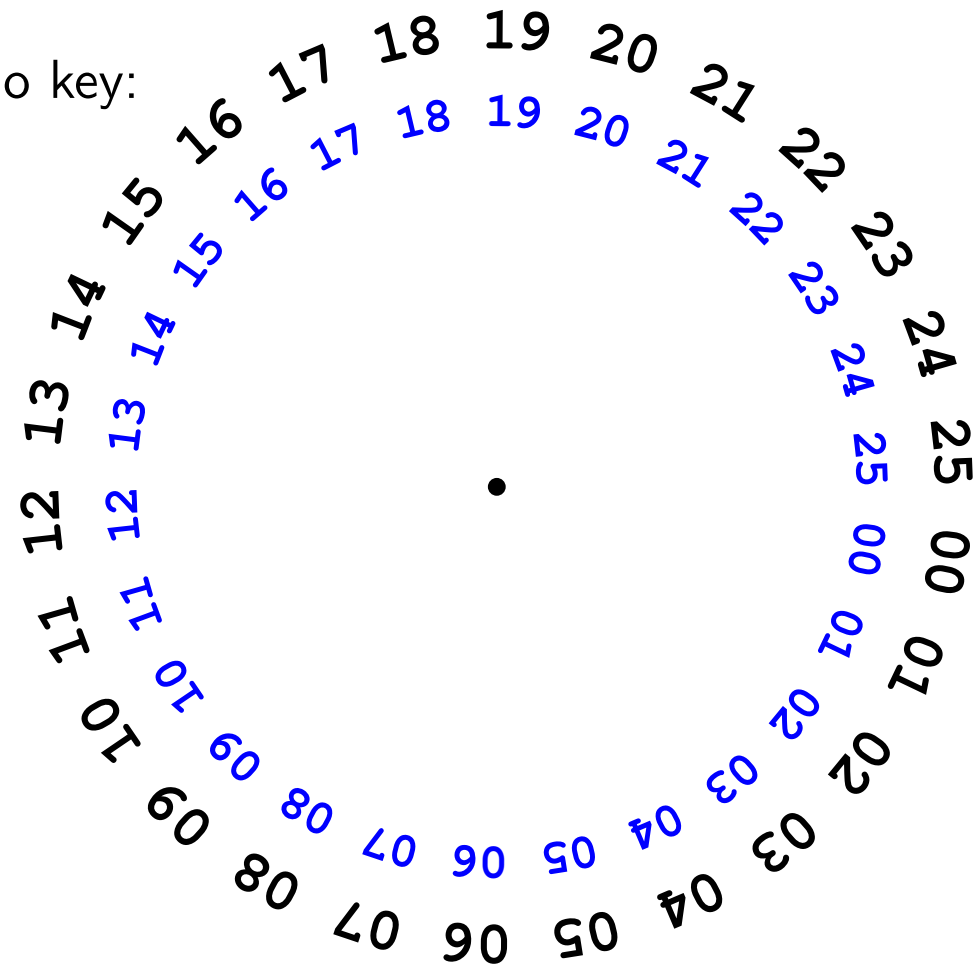

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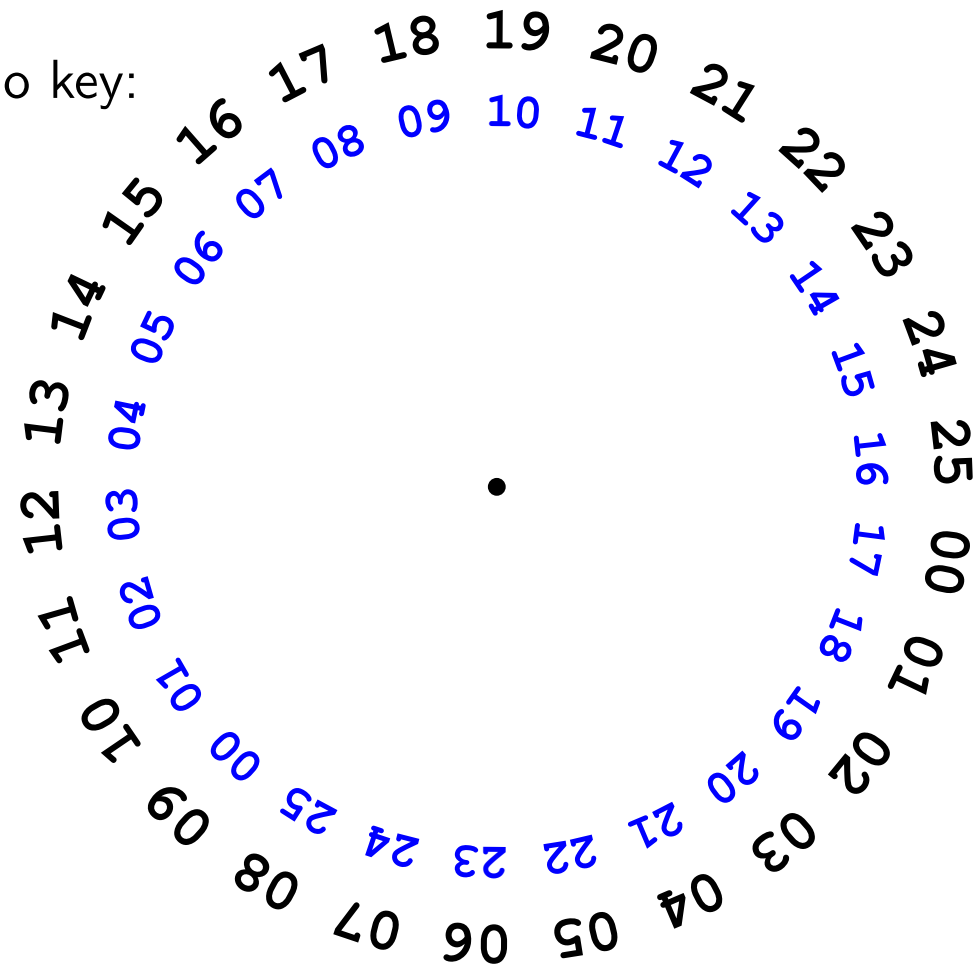

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- There are 26 different shifting distances,
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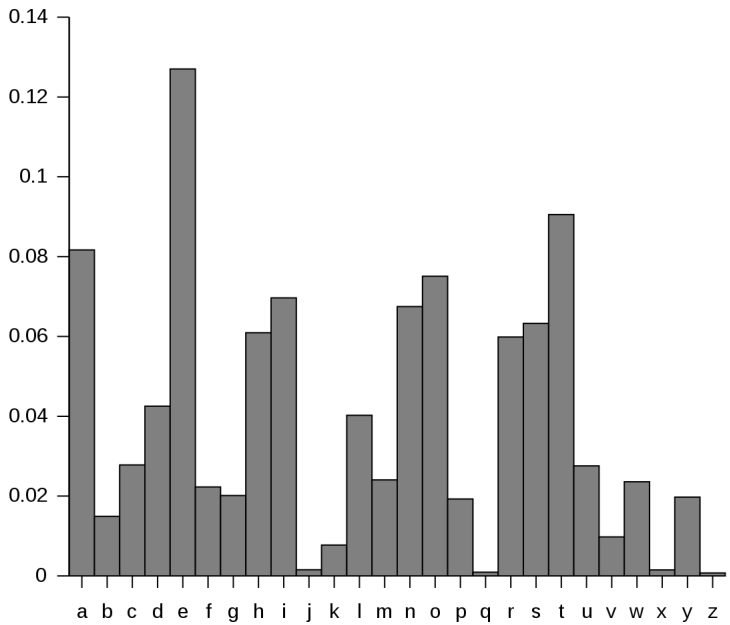
  Still possible to try on a supercomputer with some time.
- The set from which the keys are drawn is called the keyspace.
  A minimum requirement for security is that the keyspace is
  too large to search.

Cryptanalysis

For sufficiently long texts, frequency analysis is much more powerful than key search.

The frequency distribution of letters in English[1] shows a very strong peak at E, and strong peaks at T, A, O, I, and N. Can look for his pattern of peaks in Caesar to get most-likely shifting distance.



Also helps to find candidates for most common symbols in substitution cipher.

[1] Source: Wikipedia

One-time pad

- Let $m \in \{0,1\}^\ell$, i.e., a message is a string of ℓ bits.
  Let $k \in \{0,1\}^\ell$, chosen uniformly at random.
  Then $c = m + k$, where addition is done modulo 2 in each position.
  (In more mathematical notation: $m, k \in \mathbb{F}_2^\ell, c = m + k$.)
- The one-time pad is information-theoretically secure –
  there is no information about the plaintext in the ciphertext.
  $c_i = 0$ can come from $m_i = k_i = 0$ or from $m_i = k_i = 1$.
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- Possible practical usage: agree on a book to use as key;
  letter in the book determines the shifting distance of that letter.
  Using key $k =$`THISISTHESTORYOFLITTLERED..`

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
          |
TUVWXYZABCDEFGHIJKLMNOPQRS
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+ THISISTH            |
----------      TUVWXYZABCDEFGHIJKLMNOPQRS
  ALTDWTHI
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  Problem: Key is no longer uniformly distributed.

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  HELLOBOB      ABCDEFGHIJKLMNOPQRSTUVWXYZ
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Viginère

- Use codeword as key, e.g., k =CRYPTO.
  Encryption works the same as for one-time pad with key
  CRYPTOCRYPTOCRYPTOCRYPTOCRYPTOCRYPTOCRYPTOCRYPTOCRYPTOCRYPTO...

```
  THISISABETTERWAYTOENCRYPTTHANCAESAR
+ CRYPTOCRYPTOCRYPTOCRYPTOCRYPTOCRYPT
----------------------------------
  VYGHBGCSCIMSTNYNMCGEAGRDVKFPGQCVQPK
  |     |     |     |     |     |
```

  Letter 1, 7, 13, 19, ... use Caesar with A $\mapsto$ C

```
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
|       |                       |   |
C D E F G H I J K L M N O P Q R S T U V W X Y Z A B
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  |     |     |     |     |     |
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```

- Keyspace much larger than for Caesar, but for long m and known $|k|$ frequency analysis on letters $1, |k|+1, 2|k|+1, 3|k|+1, \ldots$ finds first shifting distance; on letters $2, |k|+2, 2|k|+2, 3|k|+2, \ldots$ finds second shifting distance; etc.
- For unknown $|k|$ either bruteforce length by looking for clear peaks in letter frequencies when skipping 1,2, 3, ... letters or find repeating combinations of letters; those appear likely at a multiple of $|k|$.