On the Design and Implementation of Efficient Zero-Knowledge Proofs of Knowledge

SPEED-CC, Berlin (Germany), October 13th, 2009

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Why to Avoid ZK-PoK in Hidden Order Groups

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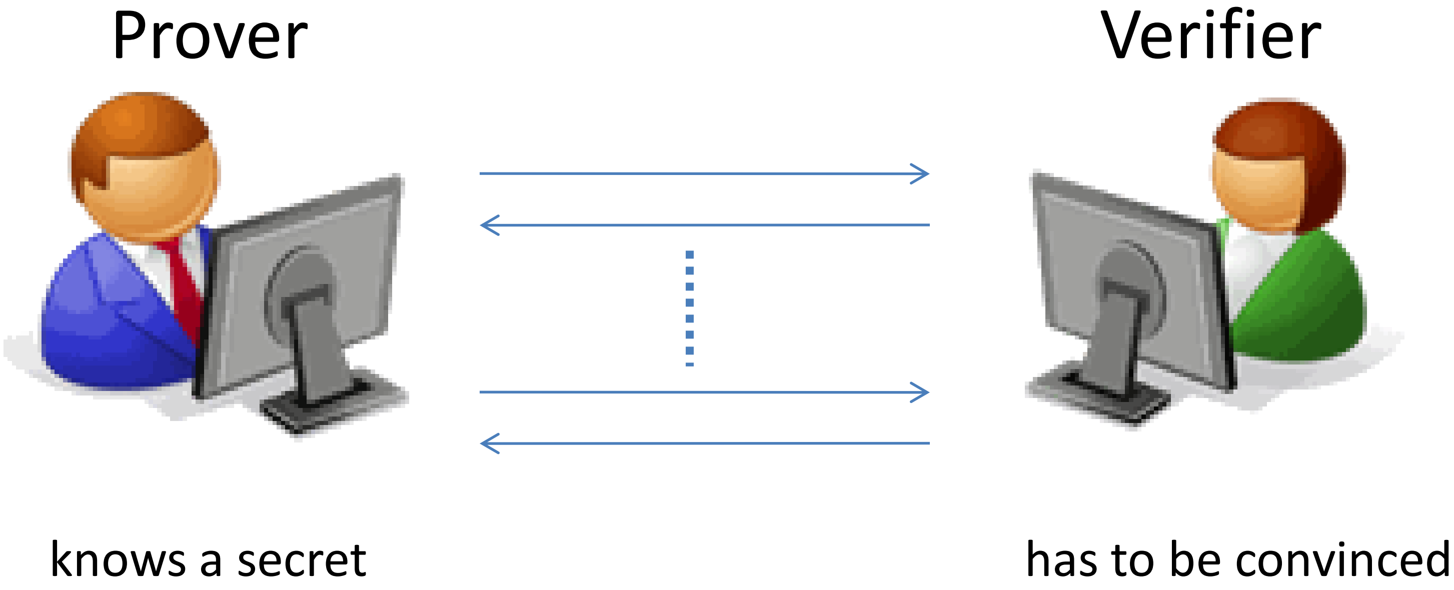
Outline

Proofs of knowledge in hidden order groups

Exact efficiency and security analysis

Conclusion

Introduction



Prover

knows a secret

Verifier

has to be convinced

Proof of Knowledge: Prover cannot cheat

Zero-Knowledge: Verifier cannot learn secret

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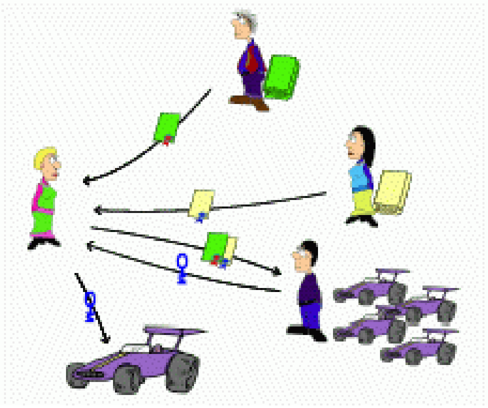
Applications

Remote Authentication

(e.g. DAA)



Credential Systems

(e.g. idemix)

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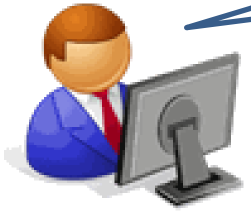
The Schnorr Protocol

I know $x = \log_g y$.



$r \in_R \mathbb{Z}$

$t := g^r$ $\xrightarrow{\quad t \quad}$

$\xleftarrow{\quad c \quad}$ $c \in_R C$

$s := r + cx$ $\xrightarrow{\quad s \quad}$ $g^s \overset{?}{=} t y^c$

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The Schnorr Protocol

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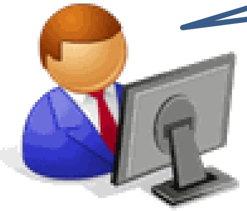
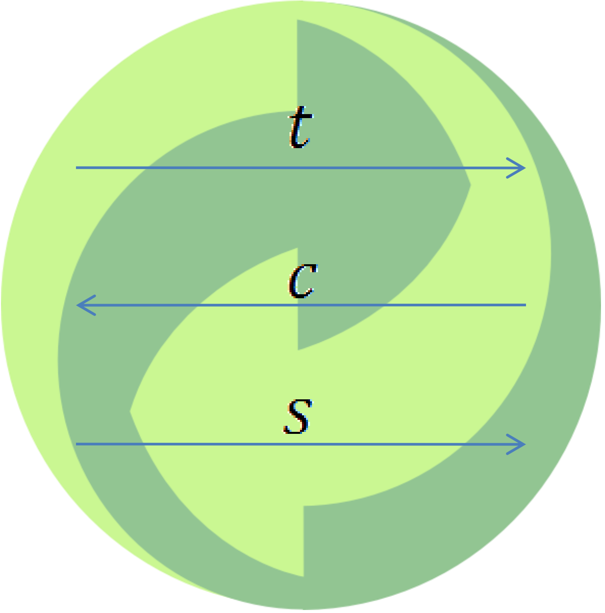


$r \in_R \mathbb{Z}$

$t := g^r$



$c \in_R C$

$s := r + cx$

$g^s \overset{?}{=} ty^c$

BUT: We must use $C = \{0,1\}$!

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A Computationally Hard Problem

Given safe RSA modulus n, and $x, y \in_R \mathbb{Z}_n^*$,

cannot compute a, b, c, w such that $w^c = x^a y^b$ and ($c \nmid a$ or $c \nmid b$).

holds under: **Strong RSA Assumption**

Given safe RSA modulus n, and $y \in_R \mathbb{Z}_n^*$,

cannot compute $a, e \neq 1$ such that $a^e = y$.

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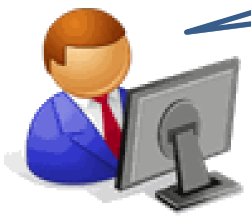
A Damgård/Fujisaki based Protocol

I know $x = \log_g y$.



$$r, \bar{r}, \bar{x} \in_R \mathbb{Z}$$
$$t := g^r$$
$$\bar{y} := \bar{h}_1^x \bar{h}^{\bar{x}}$$
$$\bar{t} := \bar{h}_1^r \bar{h}^{\bar{r}} \qquad \xrightarrow{t, \bar{t}, \bar{y}}$$

$$\xleftarrow{c} \qquad c \in_R C$$

$$s := r + cx$$
$$\bar{s} := \bar{r} + c\bar{x} \qquad \xrightarrow{s, \bar{s}} \qquad g^s \overset{?}{=} t y^c$$

$$\bar{h}_1^s \bar{h}^{\bar{s}} \overset{?}{=} \bar{t} \bar{y}^c$$

With large challenge set.

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Why it works...

$g^{s_i} = ty^{c_i}$ $\qquad i = 1,2$ $\qquad \xrightarrow{\quad t \quad}$

➔ $g^{\Delta s} = y^{\Delta c}$ $\qquad \xleftarrow{\quad c \quad}$

➔ $x = \Delta s \, (\Delta c)^{-1}$ $\qquad \xrightarrow{\quad s \quad}$

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Why it works...

$$g^{s_i} = ty^{c_i} \qquad i = 1,2$$

➔ $$g^{\Delta s} = y^{\Delta c}$$

➔ $$x = \Delta s\,(\Delta c)^{-1}$$

$$\xrightarrow{t,\bar{t},\bar{y}}$$

$$\xleftarrow{c}$$

$$\xrightarrow{s,\bar{s}}$$

$$\bar{h}_1^{s_i}\bar{h}^{\bar{s}_i} = \bar{t}\bar{y}^{c_i} \qquad i = 1,2$$

➔ $\bar{h}_1^{\Delta s}\bar{h}^{\Delta\bar{s}} = \bar{y}^{\Delta c}$ and $\Delta c \mid \Delta s$

➔ $$x = \frac{\Delta s}{\Delta c}$$

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Intuitive Comparison

Schnorr protocol:

slow

looooong

DF-based protocol:

fast

elegant

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A Closer Look

Common reference string

Only computationally sound

Bad complexity reductions

$\bar{y} := \bar{h}_1^x \bar{h}^{\bar{x}}$
$\bar{t} := \bar{h}_1^r \bar{h}^{\bar{r}}$

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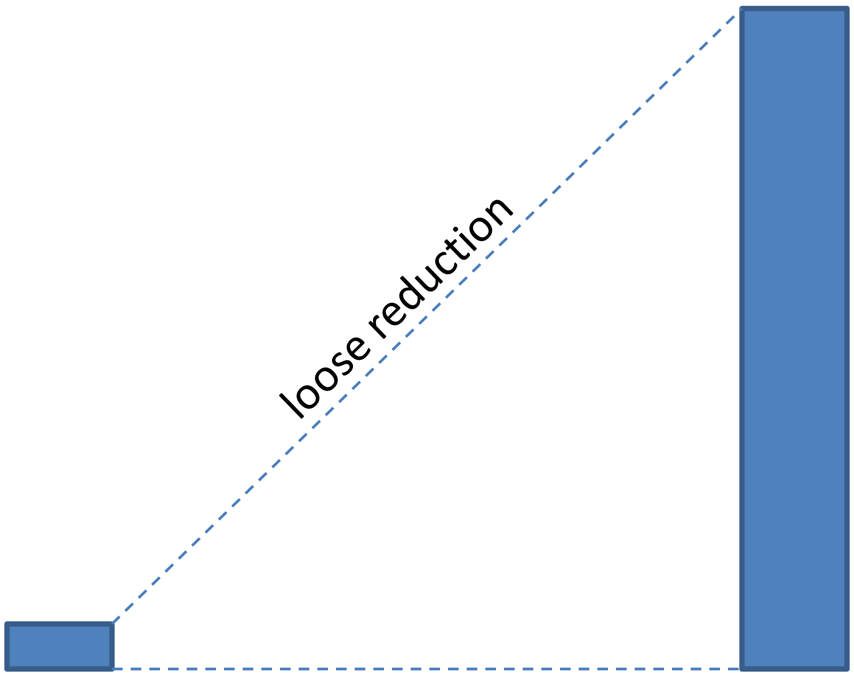
Bad Reductions



Probability of
breaking Strong RSA

Probability of
breaking the protocol

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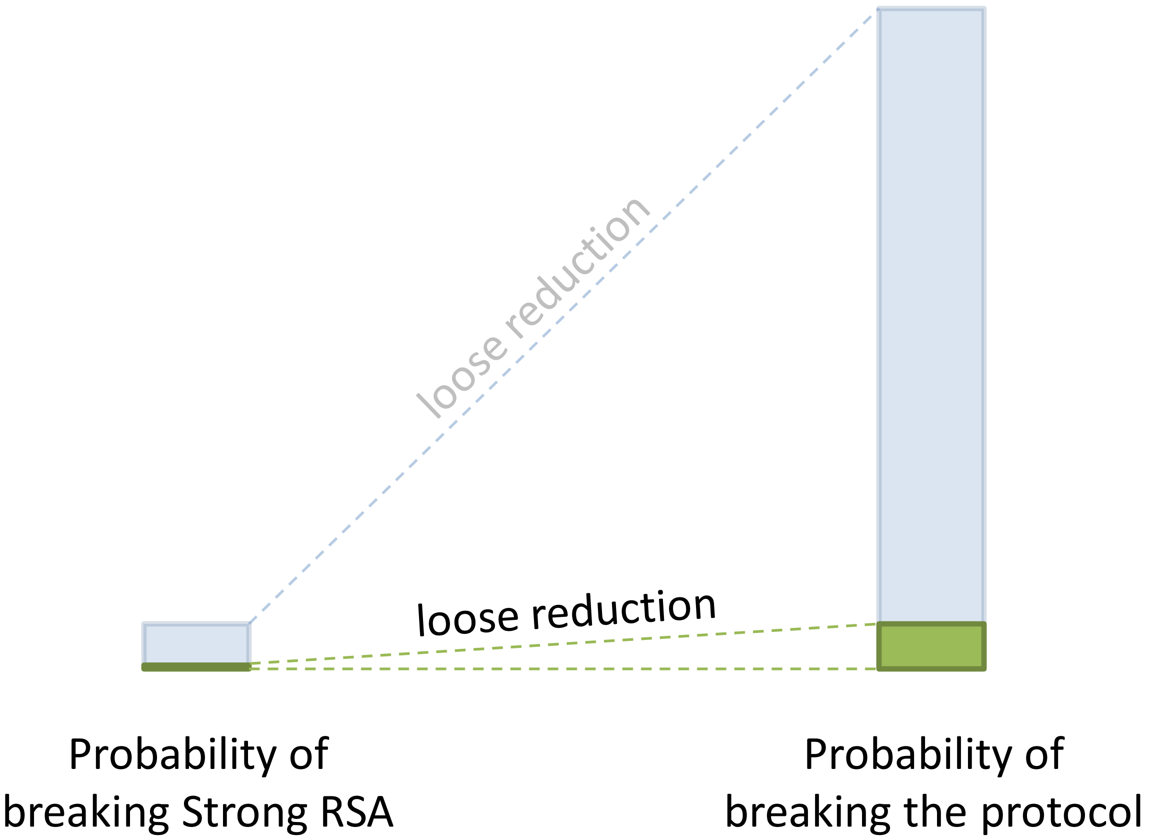
Is DAA broken?

Bad Reductions



loose reduction

loose reduction

Probability of
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Probability of
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Relative Costs

I know x, r, such that $y = g_1^{x^2} g_2^r$.

$$\frac{\text{Costs (Schnorr)}}{\text{Costs (DF-based)}}$$

for cheating probability of 2^{-80} and prover limited to 2^{80} steps.

$\lvert n_0 \rvert$	$\lvert n \rvert$ = 15528	$\lvert n \rvert$ = 2048	optimal $\lvert n \rvert$
1024	42.7	2.7	1.9
1280	24.0	1.7	1.1
1536	13.1	1.0	0.7
2048	5.6	0.6	0.3

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So…

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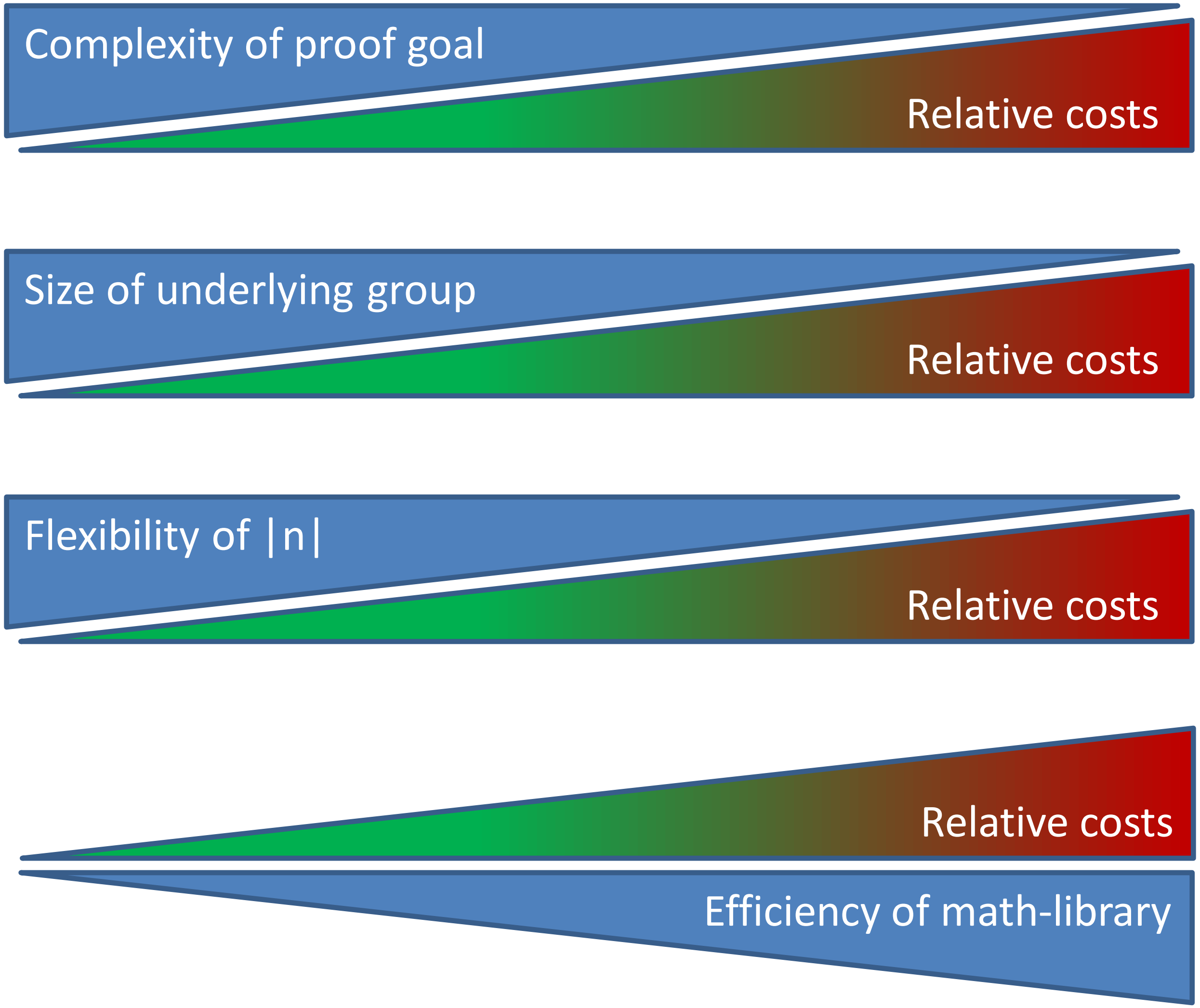
Sources of Inefficiency



Complexity of proof goal

Relative costs

Size of underlying group

Relative costs

Flexibility of |n|

Relative costs

Relative costs

Efficiency of math-library

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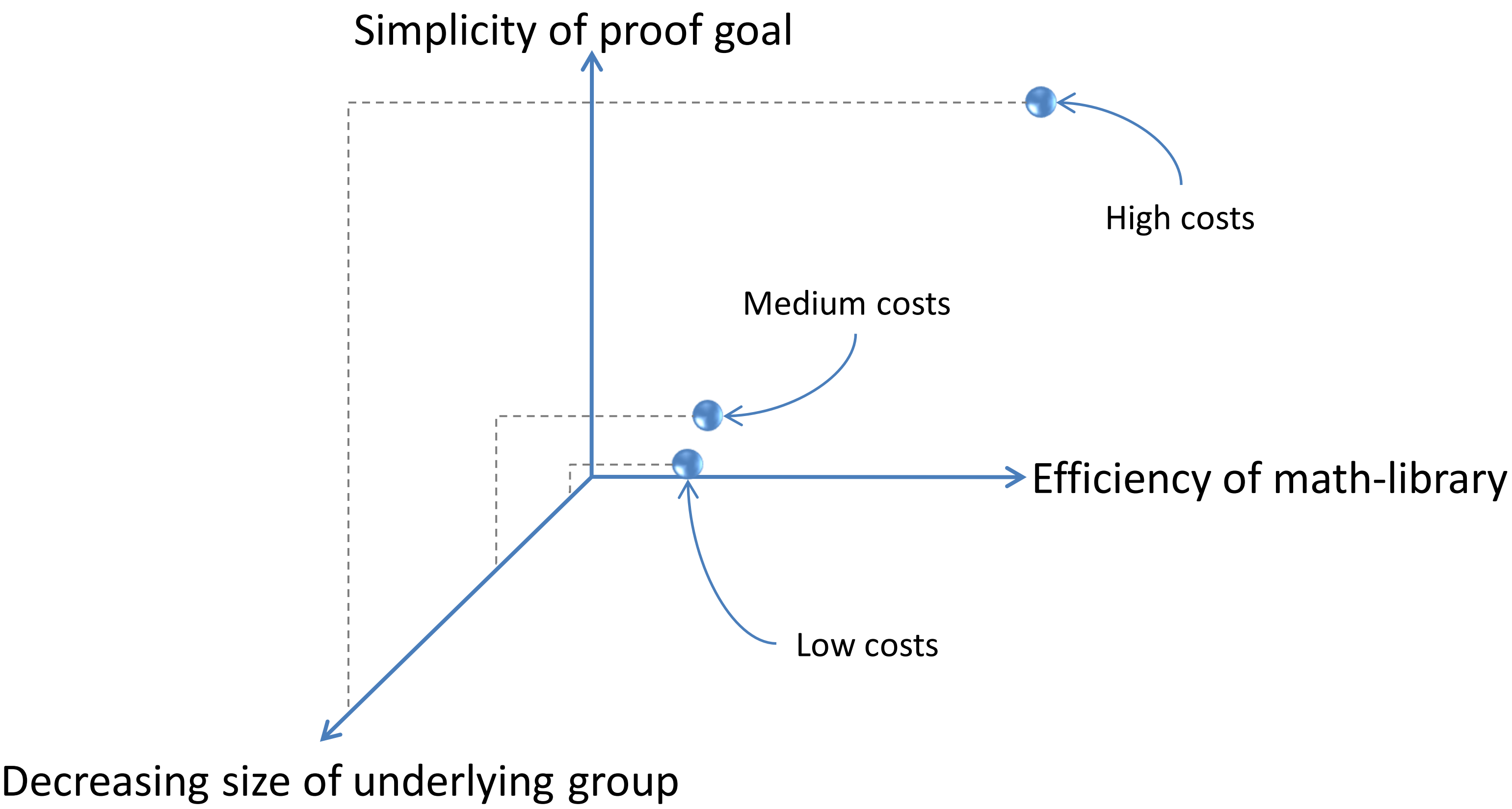
Dependencies of Relative Costs


Simplicity of proof goal
High costs
Medium costs
Efficiency of math-library
Low costs
Decreasing size of underlying group

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Crypto folklore

Design vs. implementation

RSA‘s legacy

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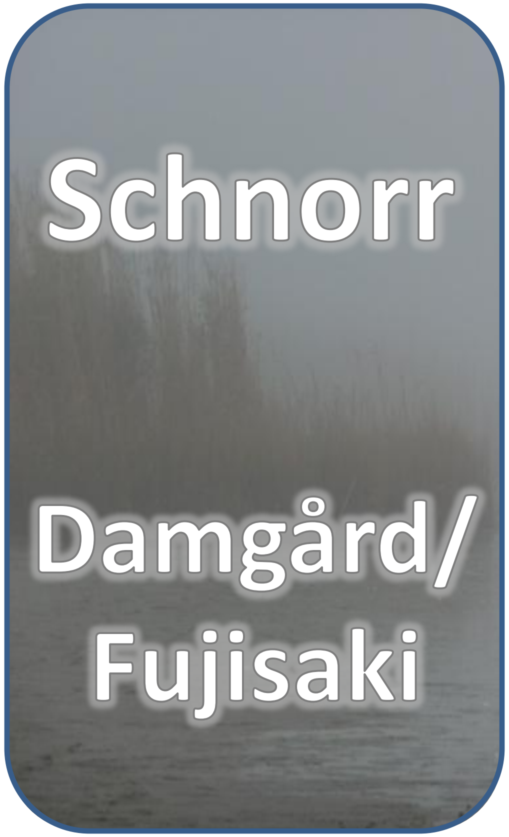
Crypto folklore



Schnorr

Damgård/ Fujisaki

Design vs. implementation

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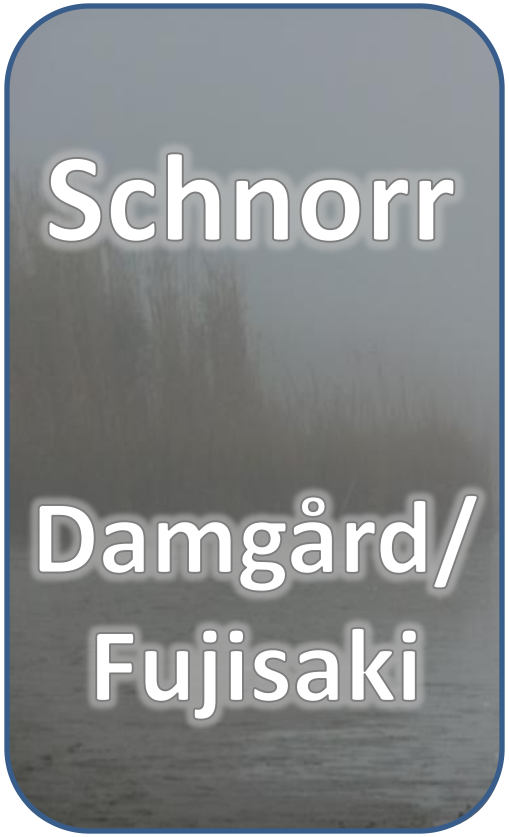

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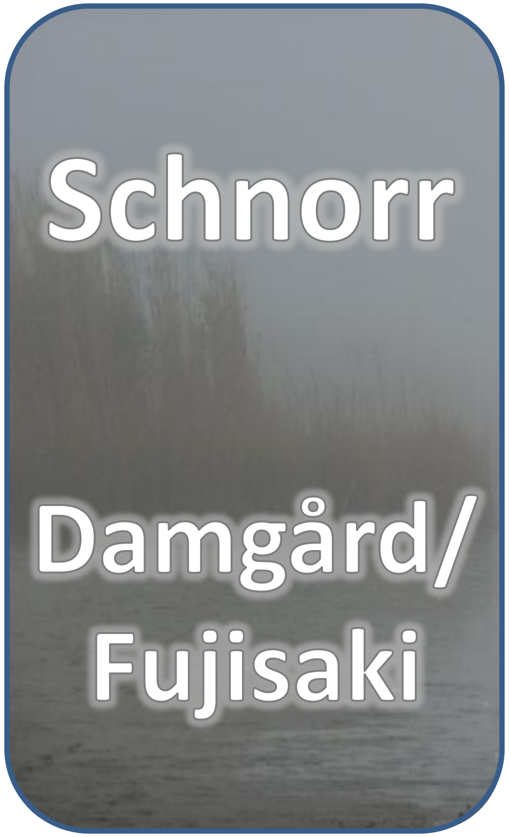

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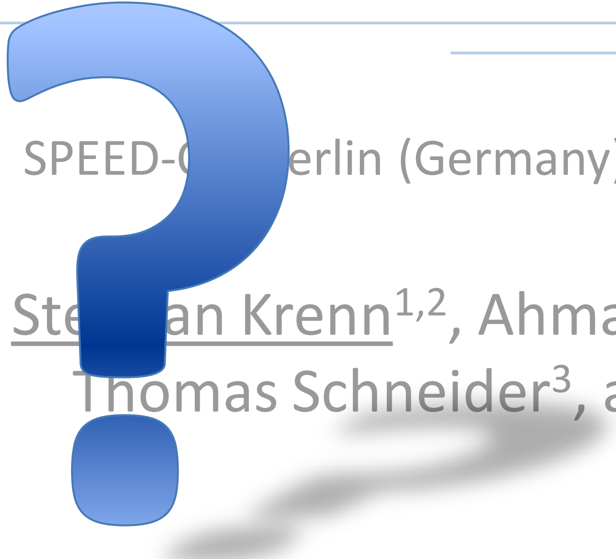

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